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ABSTRACT

This collection contains three studies of the relationship between incentives, learning, and employability. The first study, "Incentives, Learning, and Employability," by John Bishop, models students' choices concerning what to learn and how much effort to spend learning it in terms of the outcome of a comparison of benefits and expected costs and examines various strategies for use in increasing student motivation to learn. The second paper, "A Formal Model of School Reward Systems" by Suk Kang, examines pass-fail and graduated reward systems. The third paper, "The Design of Handicapping Systems for Rewarding Learning: A Mathematical Analysis of Intertemporal Issues" by John Bishop and Charles Wilson, examines alternative reward systems, a model of behavior, measures of cost-effectiveness, and student response to different reward systems. Appendixes to the third paper include the handicapping model and a discussion of the formulas used in the study; appendixes to the overall report include notes on the observational data collection procedure used, a discussion of the fair distribution rule in a cooperative enterprise, and an examination of the relationship between incentive schemes and effort level by the agents in a cooperative enterprise. (MN)

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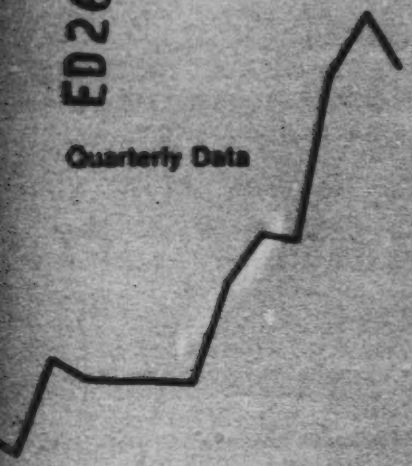

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INCENTIVES, LEARNING, AND EMPLOYABILITY

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INCENTIVES, LEARNING, AND EMPLOYABILITY

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FOREWORD

Many factors mediate the process of youth employability development. Our prior National Institute of Education-sponsored research has examined the influence of significant others, part-time work experience, and vocational training. Conditioned by the knowledge gained from analyses of the influences of these factors, and, more important, conditioned by the wide knowledge base developed from thousands of person-hours of data collection from employers through direct observation and survey techniques, our research is culminated here appropriately with a focus on secondary school experiences. The central function of high schools is learning. Thus, how and under what conditions youth learn, or fail to learn, is the focus of this inquiry. More specifically, the study addresses knowledge and skills resulting from the instructional, curricular, and contextual elements of high schools as they transfer to the workplace.

This research program was directed by Dr. Kevin Hollenbeck. The primary author of this report was Dr. John Bishop, Associate Director of Research. Dr. Suk Kang contributed to the report. Project staff who capably performed the field data collection include Joseph Copeland, Margo Izzo, Dr. Floyd McKinney, Dr. Jennifer Humphreys-Cummings, Ruth Gordon, Angela Valentine, Tina Lankard, and Tom Tinkler. Professor Kathy Borman, of the University of Cincinnati, directed the data collection at one of the sites and provided valuable training assistance to the project staff.

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Robert E. Taylor
Executive Director
The National Center for Research
in Vocationl Education

CHAPTER 1

INCENTIVES, LEARNING, AND EMPLOYABILITY
by John Bishop

1.1 Introduction

An individual's employability (perceived productivity in a job) is determined by a constellation of skills, abilities, and traits both cognitive and noncognitive. Employability development is the process of developing or learning these skills, abilities, and traits. Employability development is thus a particular type of learning. A theory of employability development is not, however, a special case of learning theory.

Theories of learning tend to take what is learned as a given. What is at issue is how much of the material--whatever it might be--is learned. However, the list of things that might be learned is endless--breakdancing, reading, car repairing, and so forth--and, not all abilities contribute equally to employability. Some abilities contribute to productivity in all jobs, others contribute to productivity in a much smaller set of jobs, and some make no contribution to productivity. There is a limit on available learning time, so choices must be made. A theory of employability development must explain these choices. Why, for example, do some people choose to learn things that make them more employable and others elect to focus on abilities that have little reward in the labor market?

A theory of employability development must also explain how much is learned. Why do some youngsters learn more mathematics than others? Why are some classroom management techniques more successful than others? Why do peers support and encourage learning in some classrooms and not in others? One simple framework will be applied to all of these questions.

Learning is a change that takes place in a person. It occurs when an individual who is ready and able to learn, is offered an opportunity to learn and makes the effort to learn. All three elements are essential.

Learning ability and readiness to learn depend on one's educational background and intelligence. Background is important because many skills and abil-

ities are hierarchical (for example, B cannot be understood until A is understood so an individual's capability to learn B depends on whether they already know A).

The opportunity to learn is in principle, open to every literate individual. For most students, however, the costs are very high if teachers and textbooks are not available. Thus, schools lower costs of learning and making school attendance compulsory lowers costs even further. When many individuals are taught the same thing at the same time, there are significant reductions in the per trainee costs of teachers. The learning opportunities, consequently, come in packages called courses or modules. Sometimes they come in large packages with sequences of required courses called programs. Educators decide what courses to offer, what to include in a course, and how much material to cover. These decisions clearly have major effects on what students are exposed to and, through this mechanism, on what students learn. How large are these effects? What happens to learning when standards are increased? What influences the effect of standards on learning? These are some of the questions to be addressed in this paper.

The part of the learning equation that is controlled by the student is effort. Within the preset constraints of the opportunity, the student, along with his or her parents, chooses which school to attend and which courses or programs to take. Students then further control their learning by regulating the effort they devote to each course. They decide whether to copy homework, whether to cut class, and whether to pay attention.[1] Studies in classrooms have found that the time engaged in learning is considerably less than the time available for learning (time not devoted to disciplinary or administrative matters). (Frederick 1977, 1979; Fischer et al, 1978; Klein et al 1979; Goodlad 1983; Halasz and Behm 1982). These studies have also found considerable variation in the amount of available classroom time that students actually devote to learning. Time devoted to homework is also highly variable.

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[1]A physical or learning disability that prevents a student from paying attention is considered to be affecting readiness and learning ability not effort. A key part of the distinction between effort and readiness is that effort is under the student's control whereas readiness is either exogenous or determined by previous decisions.

While 25 percent of sophomores report doing more than 5 hours of homework each week, another 20 percent report doing less than 1 hour per week.

Even more important than the time engaged in learning is the intensity of the student's involvement in the process. After 2 years of studying American high schools, Sizer (1984) concluded, "No more important finding has emerged from the inquiries of our study than that the American high school student, as student, is all too often docile, compliant, and without initiative" (p.54). Goodlad (1984) made a similar observation in noting, "The extraordinary degree of student passivity stands out." Student-teacher ratios of 18 to 1 imply that, in the aggregate, students spend nearly 18 times as many hours learning as teachers spend teaching. Student time is therefore a very important input into the educational process, and the intensity of use of that time a very important determinant of learning.[2]

Readiness, opportunity and effort interact to produce learning. They do so in a very special way that is represented by the following equation:

$$(1)\quad \text{Learning}_i = L_i = f[\text{Readiness}_i \times (\min(\text{opportunity}_i, \text{effort}_i)]$$

where 'i' indexes specific types of learning.

In other words, learning depends on effort (E) or opportunity--whichever is the lesser. Improvements in opportunities to learn (for example, a new course or a change in curriculum) change learning only if effort is constrained by lack of opportunity or influenced by the cost (for example, tuition) of the opportunity. Changes in effort are, therefore, not an automatic response to changes in opportunity. The second key assumption is that the readiness or ability to learn varies across people and across competencies that might be learned. This means that it is efficient from both personal and societal perspectives for people to choose different mixes and levels of learning effort.

The process of choosing learning effort. The student is not just another input into an educational process that is controlled and directed by others. Rather, students are more appropriately viewed as entrepreneurs trying to grow and develop in an environment shaped by a great variety of forces: history,

[2]Much of the impact of good teaching derives from its impact on the student's motivation to devote intense effort to learning.

parents, teachers, peers, employers, gov-ernment, society, and one's own abilities. Students generally, and secondary school students especially, should be viewed as individuals freely choosing what to try to learn and how hard to work at learning it.

It is useful to model these choices as the outcome of a comparison of benefits and expected costs. The benefits are both intrinsic--the joy of learning for its own sake--and extrinsic. The extrinsic rewards include the honor and respect that parents, peers, and teachers give for achievement, the honor and respect given to those who receive the rewards that achievement brings (awards, higher pay, better jobs, influence, and fame), and the rewards themselves. The costs are time, psychic energy, money for tuition and books, loss of control over one's in-class time, and--frequently--the necessity of giving up old habits and ideas.

The benefits of learning are derived from five different sources. Total marginal benefits are, therefore, the sum of the following marginal benefits of learning:3

(2) Joy of learning (B_L) +

+ Value of parental recognition x increased recognition by parents ($B_P P_L$)

+ Value of peer recognition x change in recognition by peers ($B_A A_L$)

+ Value of teacher recognition x change in recognition by teachers ($B_T T_L$)

+ Value of employer recognition x impact of learning on the pay and quality of one's job (Y_L)

= Benefit of increased learning (dB^*/dL)

Recognition is everything that the individual or group does consciously or unconsciously to influence the student's choices. Such influence might take the form of explicit rewards or punishment, but is more likely to assume the form of subtle nonverbal cues and expectations that are seldom verbalized. Note that the impact of teacher recog-nition depends on both the marginal value

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3The benefit of a specific level of skill can be represented mathematically by $B^* = B(L,P,A,T,Y)$. $B(*)$ is the function that describes how utility depends on the amount learned (L), recognition from parents (P), peers (A), and teachers and the school (T); and labor market payoffs (Y). Differentiating B^* with respect to L and choosing units for B^* such that $B_y = 1$, we have $dB^*/dL = B_L + B_P P_L + B_A A_L + B_T T_L + Y_L$.

that the student places on such recognition (B_T) and the degree to which teachers are aware of the learning that does occur and their propensity to reward improvements in learning (T_L). The same holds for parental and peer recognition.

The cost of learning (C) depends on two factors: the relationship between effort and learning (f_E) and the cost of effort (C_E). Both of these factors are influenced by teachers, parents, and peers. Thus, the cost of an increment in learning is given by the equation:

$$(3)\quad \frac{dC}{dL} = \frac{C_E}{f_E} = \frac{C_E(\text{opportunity, parents, teachers, peers})}{f_E(\text{readiness, parents, teachers, peers})}$$

where C_E is the marginal cost of effort and f_E is the marginal impact of effort on learning.

In this framework, students with a knack for a particular subject learn more than others because their costs of learning are lower (that is, they can learn a given batch of material in less time) and because their achievements are more likely to be recognized and rewarded by others (an absolute achievement level is the basis of almost all rewards for achievement). Significant others--teachers, parents, peers, and employers--influence what students learn primarily through their control or influence over costs and rewards. Students learn more from good teachers because..."costs"...have fallen (that is, the material is better explained enabling the student to learn it faster) and ..."benefits"...have risen (that is, the material seems intrinsically more interesting or the teacher gives praise and recognition to learning.)

Factors influencing the choice of learning efforts. Schools and teachers influence employability development in a variety of ways, including the following:

- They lower the costs of certain types of learning (for example, employment-related mathematics skills, typing, work habits) and raise the costs of learning subject matter or skills that are not a part of the curriculum. The schools control over the opportunity set (curriculum) can have major effects on what is learned.

- They directly influence the benefits of learning through the use of praise and feedback given in the classroom and the honor accorded to particular subjects to study.

- They define proxy measures of achievement (for example, tests, comments on themes, grades) and feed these back to the student

and parents. The high school transcript has a similar information transmission role for colleges and employers. Teachers' reactions to student contributions in class communicate similar information to peers. Such proxy measures can become extremely important motivators and have major impacts on student choices.

- They influence the student's perception of the labor market and societal rewards for different kinds of learning.

Peers can lower the costs of learning by studying together or teaching others and can increase the benefits of learning by recognizing and respecting others for their learning achievements. Employers influence the learning that goes on in schools by recognizing and rewarding it.

Parents are generally thought to have the greatest influence over adolescents' decisions about schooling. Parents help their children learn by lowering costs (for example, by serving as a tutor or forbidding alternative activities until homework is done) or by increasing the rewards (for example, by praising and honoring success at school). The attitudes of parents toward various forms of educational achievement are not written in stone, however. Parental attitudes respond to school policies and parential perceptions of the economic environment. Activities like interscholastic sports and sports banquets, in effect, honor the parents as well as the youth and thus strengthen parental incentives to promote athletic excellence. Plays, musical recitals, and evening assemblies recognizing academic achievement have similar effects on parental encouragement of efforts to develop these abilities. Thus, greater school recognition of learning can be expected to stimulate greater parental encouragement of learning.[4]

If academic achievement was perceived to have no effect on labor market success, parents would probably be considerably less supportive of school achievement than they currently are. Doing well in high school has a much greater impact on labor market success in Japan than it does in the United States. That is why Japanese parents place such great stress on academic achievement and often pay for 10 to 15 hours a week of private tutoring. Thus, labor market rewards for academic achievement have pervasive effects on the

[4]In other words, $P_L / T_L > 0$, which is the derivative of parental recognition with respect to school and teacher recognition, is positive.

recognition that parents give to academic achievement.[5] Students do not even have to be aware of the labor market to have their decisions influenced by it.

There is considerable evidence that the economic rewards and prestige attached to various types of academic achievement influence student decisions about continuing in school and about selecting a field of study. College enrollment and student choice of field of study respond to economic payoffs, such as the income advantage and the perceived availability of jobs in a given field (Freeman 1971, 1976a, 1976b; Bishop 1977). Labor market conditions also affect a student's decision to drop out of high school (Bowen and Finegan 1969; Lerman 1972; Gustman and Steinmeier 1981). The minimum wage (Ehrenberg and Marcus 1982) and the quality of the schooling offered (Gustman and Pidot 1973) have been shown to affect drop out rates. The financial costs of school attendance have also been shown to have major effects. Numerous studies have shown that college attendance rates are increased when colleges are nearby, tuition levels are low, and generous financial aid is available (see Jackson and Weathersby 1975 for a review of the literature).

The very simple framework outlined in the preceding section will be used to examine the following four important questions:

- How does a youth's sense of self-efficacy influence his or her decisions about employability development?
- What impacts do labor market signals have on a youth's decisions about employability development? Under what conditions are a youth's responses to these signals socially optimal? How does credentialing of educational achievement influence learning?
- How do classroom systems of recognition for academic achievement effect learning?
- How do peer group influences and systems of recognizing academic performance interact? When is it desirable to offer awards to the group rather than the individual?

Each of these questions is addressed in the following sections.

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[5] In other words, $P_L / Y_L > 0$, which is the derivative of parental recognition with respect to the labor market rewards for learning, is positive.

1.2 Self-Efficacy

The theory outlined above assumes that students choose their level of effort by calculating how much the extra effort will contribute to learning and how much specific learning will be rewarded. Since many of the rewards for learning will be realized in the future and learning is hard for the student to measure, decisions about effort must be based on anticipated rather than actual relationships. It is quite possible for beliefs to differ significantly from reality. The belief that one's effort, not fate or chance, determines one's success is called internal locus of control or self-efficacy. Numerous studies have found a positive correlation between feelings of self-efficacy and learning. Table 1 presents data from Coleman et al. (1966) concerning the verbal achievement of 9th grade students categorized by their reaction to the following statement: "Good luck is more important than hard work for success."

TABLE 1

SCORES ON A VERBAL ACHIEVEMENT TEST
BY
THE STUDENT'S SELF-EFFICACY BELIEFS

	Good Luck	Hard Work
White-North	45.4	54.8
White-South	42.9	52.5
Black-North	40.0	47.1
Black-South	36.6	43.3
Hispanic	38.6	46.2

SOURCE: Date from Coleman et al. 1966, p. 323.

The students who felt that hard work determined success had considerably greater verbal achievement scores (54.8 rather than a 45.4 for whites from the north). This relationship remained strong when family background and self-esteem were entered into the model to compete with self-efficacy in explaining verbal achievement.

Part of the reason for this relationship is that successful learning causes the student to feel more in control of his or her fate. Another reason

for the relationship is that a belief in the efficacy of effort results in greater effort and more learning. Hotchkiss's (1984) analysis of changes in test scores between sophmore and senior year found that one's sense of control over one's fate measured when the student was a sophmore had an important effect on learning. Hotchkiss also found that causation runs the other way as well. Changes in beliefs about controlling one's fate were positively affected by verbal achievement measured when the student was a sophomore.

The formal theory presented in chapter 2 predicts that when rewards are based on a pass-fail criterion, the student will give up trying to learn. Dropping out responses like these have been uncovered in laboratory research on both animals and people and are often called learned helplessness (Seligman 1972, Miller and Seligman 1973, 1975, 1976). The decision to withdraw from competition can be quite rational when the cutoff score for passing is quite high or when scores which determine whether one passes have a large random element.

1.3 Labor Market Signals

A critical aspect of employability development is to build information about the performance demands of the labor market into the incentive structure of schools and other training institutions. To do this entails a two-directional flow of information. Students must know what knowledge, skills, and dispositions are rewarded in the labor market; employers must be able to determine which individuals exhibit the characteristics that make them productive workers in specific jobs. The nature of this information flow helps to determine what is learned, how much is learned, and how well it is learned.

A complete theory of information flow between students and the labor market must answer three questions: (1) What information should be communicated? (2) What incentive structure will best motivate students to act on information they receive? (3) What rate of information flow is optimal?

The first question has to do with identifying characteristics that enhance employability. It is information about employability characteristics that should be communicated. The issue is as follows: Given the extreme variety of jobs and their corresponding tasks, what constellation of knowledge, skills,

and attitudes can best be learned at school, at other training institutions, and at work? The second question assumes as given the content of what is learned. The issue here is how to get students to learn the necessary material. The final question raises the issue of relative costs and benefits of information. Communication is costly; therefore, in determining the rate of communication, it is important to identify an optimum that balances costs against benefit.

The theoretical work presented in this report focuses on question 2. In order to show the relevance of this work to labor market signals, the theory is applied to current circumstances in the U.S. labor market. Current conditions are summarized first, then theory is used to assess those conditions.

Bishop (1985) identifies the following four key features of the connections between school performance and employment outcomes in the United States:

- Employers obtain little or no information about job applicants' grades, performance on standardized tests, deportment in high school, or participation in extracurricular activities.
- School referrals are insignificant in locating jobs for job applicants.
- High school grades and tests scores have little or no impact on the employment success of youth who have not attended college.
- The effects of cognitive abilities learned in school on employment success that do occur are not evident until an individual has accumulated several years of experience in the labor market.

Typically, the only information requested from job applicants regarding their schooling is number of years completed, the highest degree earned, and the area of specialization (if any).

Several aspects of learning theory as presented in this report suggest that these features of the information flow between the labor market and students do not generate strong incentives for learning. The most obvious difficulty is that there is no direct communication of what is learned in school to employers. As a result, there is little reward for learning in the (non-college graduate) labor market, and those rewards that do accrue do so over a lengthy time period.

Still, some trickle of information does get passed on to employers through various formal and informal channels, and some modest reward for learning is observed in the labor market. But theory suggests that the current situation is highly ineffective. There are several reasons for this. First, there is a high degree of uncertainty associated with labor market incentives for learning. Learning theory indicates that uncertainty discourages effort. Second, the importance of a diploma in certifying employability is analogous to a pass-fail testing system with a very low level of difficulty at the critical point of failure. Theory indicates that this incentive structure will give strong motivation only to those who are moderately below average in ability. High ability youth can pass (earn a diploma) with minimum effort, and very low ability youth are motivated to drop out because their chance of success is too small. Third, labor market payoffs to learning accrue so slowly that individuals with strong preferences for income now rather than later see little reason to exert effort in learning. Since many high school students probably have relatively short time horizons, labor market payoffs provide poor motivation for them. Finally, the incentives in the labor market are analogous to school evaluations in which all students are judged against the same criteria. Thus, the advantages in motivating students that occur when each student is evaluated against his or her own baseline ability are clearly not present in the labor market.[6] In summary, the theory developed in this report suggests that the U.S. labor market provides a poor incentives for learning in our nation's schools.

1.4 Student Reward Systems

Motivating students to try harder is one of the primary objectives of classroom reward systems. Classroom rewards are sometimes graduated (for example, a numerical grade on a test) and sometimes discrete (a passing grade versus a failing one). Which of these forms of feedback and reward is more effective at stimulating effort? If a pass-fail system is used, where should the passing criterion be set? Which system is least likely to induce a student to drop out?

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[6]To a degree, the concept of matching individuals to jobs could be viewed as a handicapping system, but this is a rough analogy at best.

Recognition can be based on an absolute measure of performance, on progress since a previous test, or on performance relative to an individualized target (handicap). Which system stimulates effort the most? Which system minimizes the propensity of the least able students to give up on the task of learning (to dropout)? If handicaps are used, how should they be formulated?

Rewarding progress or rewarding achievement. We begin by examining whether recognition should be given for an absolute level of achievement or for growth over an individualized baseline performance level. The primary purpose of recognizing and rewarding learning achievements is to induce students to achieve all that they are capable. Most of the recognition for academic achievement, however, goes to a very few students who are at the very top of some absolute scale of achievement. Although such rewards may challenge the most talented students to achieve, they are out of the reach of most students and hence do not influence their effort. In fact, in many schools, the majority of students see these prizes and the values they represent as threats to their own self-esteem. Their reaction is often to denigrate the reward and the learning it represents and to honor other forms of achievement--athletics, break dancing; or being "cool," attractive, or popular--which offer them better chances of success. In some cases, the denigration of learning takes the form of cheating or acting up in class and thus, may disrupt the learning of others.

Teachers sometimes try to deal with this problem by praising students who try hard and by withholding praise from students who do not. There are limits to this approach, however, for teachers are not able to observe the true intensity of a student's effort and, as a result, may reward only the visible proxies of effort. In addition, peer group norms often stand in direct opposition to the teacher's norms. Often, peer norms call for getting by with a minimum of effort; therefore, being singled out by a teacher as someone who tries hard may hurt one's reputation (Covington and Beery 1976). As Thomas (1980) has put it, "Low effort has great survival value for many students. It provides an excuse and a way to avoid ascriptions of low ability under failure conditions and can serve to aggrandize estimates of ability when success occurs" (p. 215).

There are really only two kinds of reward structures that avoid these problems. The first system organizes competition so that it occurs among students of approximately equal ability and makes awards to the winners in each league.[7] The second system recognizes and rewards the achievement of individual learning goals tailored to the student's ability and background. The individualized goal must be achievable, but not be so easy that the student does not have to make an effort to achieve it.

In chapters 2 and 3 of this report we present a formal analysis of reward systems that recognize achievement in excess of an individualized learning goal. The main findings of the analysis are summarized and explained in this chapter without the use of mathematics.

The assumptions that lie behind the analyses are the following:

- Learning requires effort.
- Effort is costly.
- The more a student tries to learn in a given time period, the greater is the marginal cost of additional learning. This occurs because there are diminishing returns to increases in effort and because the marginal costs of effort increase as effort increases.
- The test that is the basis for distributing rewards should measure the skill and knowledge that are the objectives of the course. The test need not be a perfect measure of achievement of learning objectives; however, it is important that errors in measurement be random and not too large. (If the test measures only some of the learning objectives and students are aware of this, rewarding performance on the test will tend to shift student effort toward only those learning objectives that are included on the test.)
- Students have beliefs about the magnitude of the average relationship between learning, effort, and the cost of effort and about the variability of test scores given their effort. Test score outcomes have a random element partly because the tests are not perfectly reliable measures achievement of learning objectives and partly because students are not able to anticipate exactly how much learning their effort will produce.

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[7] In team sports, this restructuring of competition is common. High schools of the same size typically compete against each other in state tournaments and extra mural softball and soccer leagues are commonly organized into divisions.

Not surprisingly, the model predicts that an increase in the reward for learning will generate an increase in effort. The model also predicts that combinations of rewards for those who learn the most and penalties for those who do not learn also increase effort. Our focus is on the effects of other environmental factors such as test reliability, feelings of self-efficacy, and reward structure; therefore, the analyses is conducted under the assumption of a fixed budget for rewards and no penalties. A somewhat less obvious prediction is that the reliability of the tests used to measure learning (and student beliefs about the importance of chance in determining test outcomes) can have important effects on the effort a student puts into learning. If rewards are based on a pass-fail criterion that has been set at a point designed to elicit maximum effort from the student, less-reliable tests will reduce the degree to which success (passing) depends on effort and thus cause effort to be reduced. The same result can occur when the amount of reward is scaled to performance in a linear fashion (as with grades and numerical test scores), if students are risk averse, or if there are floors or ceilings on the linear reward system. This implies that teachers may increase the effort that students put into their studies by using the most reliable tests possible and by convincing students that effort, not chance, determines their test score.

Another issue addressed in the analysis is the setting of standards. Assuming that the error in measuring performance is normally distributed and that the costs of an extra unit of learning are increasing at a constant elasticity, the greatest amount of effort is elicited when each student perceives about a 60 to 90 percent chance of passing provided he or she puts out the optimal level of effort. Up to a point, raising the criterion for a passing grade raises the effort that students put into learning. For the assumptions specified in table 2 (see chapter 2) the increase in learning is equal to about two-thirds of the increase in the passing standard. This means that the failure rate rises as the passing standard increases, but not by nearly as much as one might anticipate if one assumed effort does not respond to the level of the passing standard. These results depend on how rapidly the marginal costs of learning rise and on the variance of the test score.

One of the most important findings of the analysis was that pass-fail reward systems have very different effects on the amount and distribution of learning than do graduated reward systems. When rewards are a linear function

of learning, the faster learners experience a higher payoff to learning than slow learners and so they put out more effort than slow learners. While slow learn-ers put out less effort they always put some effort into their study. In a pass-fail reward system, a single passing standard for everyone has the effect of inducing the moderately slow learners to work harder than the fast learners but it causes the very slowest learners to drop out all together.

Establishing a pass-fail system that has different passing standards for students with different readiness and ability can forestall dropping out and can induce everyone--slow and fast learners--to try harder. Graduated reward systems that reward performance above an individualized learning expectation are also more efficient than graduated reward systems that reward all learning above a threshold of zero.

The empirical evidence. Four experiments have been conducted to evaluate the effect of classroom reward systems based on individualized learning expectations (ILE). In three of the four studies, there were large positive effects on student learning.

In the first experiment, Slavin (1978) used a 2 x 2 factorial design to study the effects of team versus group rewards for performance and the effects of comparison of students within ability-homogeneous groups versus comparison of students with the entire class. The theory outlined previously suggests that competition elicits the greatest effort when competitors are evenly matched (Lazear and Rosen 1981). Consequently, when students compete for individual rewards, it is important that they be evenly matched. When teams compete for group rewards, it is important that the teams be evenly matched. When team scores are an average of individual scores, it should not matter whether scores are scaled by comparison with the entire class or scaled by comparison with other students of similar ability. If the competing teams are evenly matched, winning depends on everyone's trying, regardless of how scores are scaled. Consequently, one would expect the nature of comparisons (whether it is with equals or with the entire class) to affect performance only when recognition is based on individual performance.

A total of 205 seventh-grade students (all but 3 of whom were white) in 8 English classes participated in the experiment. Four of these classes did not group students into teams, but instead rewarded students for their individual

performance. Rewards went to the winners of a competition among students of relatively equal ability in two of these four classes. A weekly class newsletter recognized those who had done well in their divisional competition for high scores. Students received standard percentage scores on their quizzes in the other two classes. The experiment manipulated a supplementory reward system (recognition in a class newsletter); it did not change the formula for calculating report card grades.

The divisional competition and the newsletter announcing the results raised time on task from 73 to 82 percent, a statistically significant increase. It did not, however, produce a statistically significant improvement in performance on either the standardized or curriculum-specific achievement measures.

The second experiment (Slavin 1980) involved 10 teachers and 16 English classes containing 387 sixth, seventh, and eighth graders. Whenever possible, teachers taught both experimental and control classes, and random assignment was used to determine which classes were assigned to the ILE treatment. All students studied the same 9-week language mechanics unit and covered the material on the same schedule. The only difference between treatments was in the use made of the students' scores on biweekly quizzes. In the ILE treatment, a baseline score was established for each student based on a 50-item pretest. The base score was set 10 points below the students' expected summed scores on the two weekly quizzes, and bonus points were awarded for exceeding the base.[8]

Achieving the base score or exceeding it by 1 or 2 points earned 1 bonus point. Exceeding it by 3 to 5 points earned 2 bonus points and so on. Achieving a perfect paper or exceeding one's base by 18 points or more earned a student a maximum of 7 bonus points. New base scores were calculated each week to reflect more recent performance and to correct for possible unreliability of the pre-test. The new baseline score was a weighted average of the latest score (wt = 1) and the last week's baseline plus 10 (wt = 3) minus 10. Slavin described the experiment as follows:

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[8]The problem of variability in test difficulty from week to week was handled by adjusting all quiz scores to a standard class median of 40. If, for example, the true class median was 32, 8 points were added to each score.

> Students were told that their task was to exceed their base scores by as much as possible. Each week, students were informed of their base scores, the degree to which they exceeded them, and their [plus] bonus points. A weekly class newsletter listed the students who earned 5, 6 or 7 plus points. . .
> In the control classes, students studied the same material on the same schedule but they simply received grades on their quiz scores. They did not receive base scores, plus points or newsletters . . . All students . . . received 6-week grades based on their actual quiz scores (not their plus points) and their other English-related activities, such as literature and composition. Thus, the reward system used in the experimental group was a supplement to, not a replacement for, traditional grading (Slavin 1980).

This rather mild intervention produced a statistically significant increase in student performance on the posttest that was equal to 42 percent of the pooled class-level standard deviation for the posttest. This experiment demonstrates that a remarkable gain in achievement can be obtained by simultaneously changing the way students receive feedback on their performance (emphasizing plus points in quiz grading rather than raw scores) and giving greater recognition to successful learning (publishing a newsletter).

Since it might have been the newsletter rather than the ILE grading that caused the improvements, more experiments were necessary. The third experiment (Beady, Slavin, and Fennessey 1981) involved an ILE system similar to the one described previously, except that (1) the baseline recalculation was somewhat more sensitive to recent performance, (2) there was no adjustment for week-to-week variations in the difficulty of the quiz, and (3) the seven top performers were recognized by certificates rather than through a newsletter. Beady and his co-workers described the procedure as follows:

> The comparison group studied the same math curriculum using the same focused schedule of teaching, worksheets and quizzes . . . However, instead of receiving points in comparison to base scores, these students received individual percentage scores, and the seven highest scoring students received the same certificates given the ILE students (Ibid 1981 p. 16).

This intervention produced a slightly smaller gain (about 30 % of a standard deviation) on the posttest. Even though this gain was quite respectable, it did not represent a statistically significant difference from zero because the sample size (6 classes and 180 students) was considerably smaller than in the previous experiment.

Beady, Slavin and Fennessey (1981) interpreted the lack of a significant effect as not supporting the findings of Slavin (1980) regarding the beneficial effects of basing rewards on improvement rather than on relative standing, and they spent a number of paragraphs speculating on why there was a difference between the two studies. In fact, however, the estimated effects of ILE in the two experiments are not significantly different. If the initial hypotheses had been that effects were equal to 42 percent of a standard deviation, it would not have been rejected.

The subjects of the fourth experiment were 248 seventh- and eighth-grade students in 10 mathematics classes in a desegregated Baltimore middle school. The treatments were identical to those used in the 3rd experiment described previously. Both the ILE classes and the control classes used a focused schedule of instruction for 10 weeks. Both received certificates; however, in the ILE class, the certificates were based on improvement whereas, in the control group, they were based on relative standing in the classroom. The ILE treatment was found to have a particularly positive effect on blacks, and for this group, the treatment was statistically significant. The gain in achievement attributable to ILE was about 24 percent of a standard deviation (on the posttest) for blacks and about 12 percent of a standard deviation for whites and blacks together.

Slavin (1983) has concluded that "use of equal opportunity scoring may enhance the achievement effects of cooperative learning, but the evidence is not yet conclusive," (p. 72). We agree with this assessment when students are grouped in heterogeneous teams and are rewarded for group performance. When equally matched groups compete every team member's performance counts; therefore, motivation is strong even when individualized learning expectations are not used. ILE appears to have important (20 percent of a standard deviation is important in our view) effects on learning when rewards go to individuals rather than teams.

The ILE reward systems tested in these experiments, however, had updating rules which made the baseline very sensitive to one's performance in the last few weeks. In chapter 3, we show that pure ILE reward systems with an updated ILE invite students to lower their baselines by purposely doing poorly at first and then surprising the teacher with a great improvement. If all rewards are based on the ILE, the down-up strategy would be the way for most students to

maximize rewards over the course of the experiment. Quiz scores were probably a component of the final grade in most of the ILE classes, so it is unlikely that students would do poorly on purpose. Nevertheless, the rapid updating of the ILE baselines does significantly reduce the incentive to do well on the quizzes in the early part of the marking period.

The rationale for updating the ILE on the basis of later quiz scores is the need to improve the accuracy of the student's ILE. The test given at the beginning of the year that is used to construct the ILE is not a perfectly reliable measure of the student's ability, so the initial ILE is set too high for some students and too low for others. This initial classification error is eliminated over time by incorporating additional quiz scores into the ILE. The result is an ILE that is a more accurate reflection of the student's ability and an ILE will as a result be seen as fair. This is a legitimate argument for updating the ILE, but is is also an argument for greater care in assigning the initial ILE. Initial ILE's should not be based on one pre-test. The teacher should obtain the student's scores on aptitude tests and standardized achievement tests in the subject (or a related subject) being taught, and grades in the same subject in previous years. In addition, the first class period of the year should be devoted to a 50 minute placement exam that covers both the material from previous years and the material scheduled to be covered in the class. The test scores and grades could then be combined into a single index by calculating Z scores and then constructing an average.[9] With more accurate initial ILE's, the updating of the ILE's on the basis of recent quizzes, could be postponed until the end of the first or second month and the sensitivity of the ILE to recent quiz scores reduced by reducing the weight assigned them in the calculation of the ILE.

The analysis predicts that an ILE system that uses a reliable ability test to establish the initial ILE for each student and then sticks with that baseline (or adjusts very slowly to more recent performance) will generate considerably larger increases in learning than an ILE that is rapidly updated. Experiments that contrast an updated ILE with a fixed ILE would be very helpful.

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[9]Another approach to normalizing the scores so that they can be combined involves calculating the means of the top and bottom quartile, dividing the score or grade by the difference betwen the two means, and then averaging them.

1.5 Peer Influences

A respected creative writing teacher in a highly-rated school district recently advised a 6th grader not to volunteer to answer questions when classmates did not volunteer. The explanation he offered was that volunteering too often would make the student unpopular.

Why are the students who study hard and excel in class often rejected by their peers? One reason, already discussed, is that since more recognition is limited to a few, most students conclude they cannot be winners in the academic arena. Consequently, they set other goals for themselves and denigrate academically successful students.

A second reason is that most classrooms are competitive environments in which one student's success reduces the chances of others. Grades are based on a curve. Awards are given for rank in class rather than for achieving a fixed standard of excellence. If everyone in the class were to improve, the group as a whole would not benefit. This is because the total number of awards is fixed.

Adolescents care a great deal about their reputation with peers. To them, the rewards teachers offer--grades, a smile, verbal praise, a project pinned upon a bulletin board--seem small in comparison with the rewards that their peer group is able to bestow--friendships, social acceptance, and popularity. Peer norms that reflect the interests of the group develop naturally. Actions that bring honor to the group as a whole, such as outstanding performance in an athletic competition, the orchestra, or a play, are respected and heavily rewarded by peers.[10] Efforts to win a zero-sum competition such as studying hard for a test or volunteering in class are actively discouraged by peers. Activities which do not affect one's immediate peers, such as studying for an SAT or a statewide Regents exam, reading War and Peace for the fun of it, or competing in a science fair receive better treatment from peers; however, peer pressure still works against such activities because they compete with peer group priorities regarding the use of time.

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[10] In one school we visited, for instance, each member of the football team received a visit and a present of cookies or something similar from a member of the pep club the night before each game.

How can schools and teachers get peers to value academic effort and learning? One possible approach is to reward the group for the individual learning of its members. This is the approach taken in cooperative learning. Students are grouped into evenly matched teams of 4 or 5 members that are heterogeneous in ability. After the teacher presents new material, the team works together on worksheets to prepare each other for the periodic quizzes. The team's score is an average of the scores of team members, and high team scores are recognized in a class newsletter or through group certificates of achievement.[11]

Slavin (in press) has recently reviewed 27 field experiments that compared cooperative learning strategies combining group study and group reward for individual learning with the standard individual-reward-for-individual-learning system (Slavin 1985).[12] In 24 of these studies, cooperative learning had a statistically significant positive effect on learning. Where effect sizes were available, they were approximately 30 percent of a standard deviation on the posttest.

A number of studies have been conducted in which the various components of the cooperative learning model described previously have been tested on their own or in 2 x 2 factorial experimental designs. The four studies that examined the effects of group study without group rewards for individual learning found that such a strategy had no positive effects. Group study methods that offered group rewards based on the quality of a group product were also not found to increase learning. These results suggest that the two key ingredients for successful cooperative learning are as follows:

- A cooperative incentive structure--awards based on group performance--seems to be essential for students working in groups to learn better.
- A system of individual accountability in which everyone's maximum effort must be essential to the group's success and the effort

[11] In many cooperative learning systems, the individual's contribution to the team score is a gain in score relative to an individualized learning expectation.

[12] The review was limited to studies in which treatments lasted at least 2 weeks in a regular school setting. The experimental and control groups were exposed to the same curriculum, and students were not allowed to help each other on final tests.

and performance of each group member must be clearly visible and quantifiable to his or her group mates.

These results provide important evidence of the importance of peer norms. What seems to happen in cooperative learning is that the team develops an identity of its own, and group norms arise that are different from the norms that hold sway in the student's other classes. The group's identity arises from the extensive personal interaction among group members in the context of working toward a shared goal.* Since the group is small and the interaction intense, the effort and success of each team member is known to his other teammates. Such knowledge allows the group to reward each team member for his or her contribution to the team goal, and this is what seems to happen. This is the prediction of the theoretical examination of cooperative organizations presented in appendix C, which addresses the issue of how a cooperative group will distribute the rewards it receives to its members when the sum of the individual efforts of its members determines the magnitude of the reward received by the group. The conclusion of the analysis is that the sharing rule adopted by the cooperative organization will take into account the effort and contribution the member made to the group goal. When the output is a simple sum of individual outputs (as in cooperative learning without task interdependence), there is a one-for-one passing through to group members of the reward contingencies faced by the group.

Our interviews with students on cooperative learning teams suggest that this prediction of theory is correct. Team members do encourage each other to try harder. One 9th-grade boy who found math difficult said the following:

Student 1: He [the teacher] gives me all these problems and they [team members] say you better get them right and I can't.

Interviewer: So what happens when you say that [I can't do the problems]?

Student: They tell me not to give up.

Interviewer: How does that make you feel?

Student: Makes me want to put more effort into it.

Another student was asked if he had had an opportunity to teach or help anyone in his group. He answered as follows:

Student: Yes, I've helped some people but some people start to give up and that doesn't help.

Interviewer: What do you say to a team member when they want to give up?

Student: You just can't, we want to do good on our test and all and bring our points up and all.

The passing through to the individual of the incentives facing the group can explain why cooperative learning does no worse than individual reward for individual learning, but it cannot explain why it does better. Three possible explanations are as follows:

- Group study motivated by a group reward is a more efficient method of learning than individual study.
- When rewards go to the group and groups are evenly watched, every student now has a chance of being a winner, so students do not drop out of the competition.
- The total amount of rewards available for learning have increased in the group study environment.

Group study is more efficient. The interaction that occurs within a cooperative learning team can be thought of as reciprocal tutoring. The motivational dynamics are similar as well. In tutoring, each party can observe the effort and success of the other and, as a result, each is accountable to the other. In addition, the progress of the tutee is often being monitored, so the tutor's teaching success is often visible to others. Reviews of research on tutoring have found that it has large positive effects on the learning of both the tutee and the tutor. These positive findings suggest that the tutoring interaction induced by cooperative learning may be one of the reasons for its positive effects on learning. The failure of group work alone to have a positive effect on learning, means that a cooperative reward structure is absolutely essential for students to really commit their energy to the tutoring interaction.

The interviews revealed that cooperative learning does induce students to take responsibility for how much their team mates learn. The following conversation illustrates how many of the students felt:

Interviewer: What do you feel are your committments or responsibilities as a group person?

Student: Sometimes when they don't understand, I try to explain because if its going to hurt them its going to hurt the team.

Interviewer: How do you give each other encouragement and praise?

Student: I receive praise every day. It makes you feel good and want to do better.

Interviewer: Are you being as successful in your other classes?

Student: No, I am in English.

Interviewer: What do you think makes the difference?

Student: The team.

Interviewer: Have you always been successful in math?

Student: I can work with numbers real easy. Because I like math and math relates to science.

Interviewer: What do you do when your team mates don't do well?

Student: I get upset, if I don't do well than I have no reason to get upset but if I do well. Then I know I haven't taught them. Or they are not listening.

Another reason why team learning may be more efficient than traditional learning is that students find studying to be more fun because it is now a social undertaking. One female student in a cooperative class told us the following:

Student: I am not the kind of person who can sit down for a long time. I must talk and socialize. The class is very casual. Learning is fun in team class . . . You don't feel as if you're learning in class.

Interviewer: Do you feel that team learning is a better way to learn?

Student: Yes it is because its funner (sic). Its something to look forward to at the end of the day. It makes you want to learn.

A male student said the following:

Student: I think cooperative learning has helped me a lot . . . Plus it's the teacher and how the teacher teaches. How much fun you put into the class.

Interviewer: Why this way of learning?

Student: Because you have kids helping kids plus this way makes it more interesting.

Every student has a chance of being rewarded. In cooperative learning, this characterictic is a function of the fact that competition is between evenly balanced teams. ILE scoring is often a part of cooperative learning, but with evenly balanced teams, it may not be necessary. Evidence that the "everyone-can-be-a-winner" effect is part of the reason for cooperative learning's success is provided by the four experiments discussed in section 1.4 where rewards based on ILEs were implemented in classes that rewarded individual learning. Important positive effects were found. This suggests that cooperative learning would probably not work as well if the teams were not evenly matched or if ILE scoring was not present.

Total rewards have increased. Cooperative learning is a supplementary classroom reward system. Students continue to receive grades based on individual performance. Probably one of the important effects of cooperative learning is that it significantly increases the total amount of recognition given to student learning. Newsletters and certificates recognizing team accomplishments are always a part of a cooperative learning experiment but are seldom a part of the control environment. Brophy (1981) has observed, "classroom-process data indicate that teacher's verbal praise cannot be equated with reinforcement. Typically such praise is used infrequently, without contingency, specificity or credibility" (p. 5). Cooperative learning may change the way teachers use verbal praise but even if it doesn't, it significantly increases the amount of attention that is given to learning success. These effects of cooperative learning on the total amount of recognition being given students is illustrated by the following conversation.

Interviewer: Before your were in the cooperative class, were you rewarded for working hard?

Student: No.

Interviewer: Were you embarrassed when you first went into his class and started receiving praise?

Student: No, I liked it. (laughter)

Interviewer: Did you?

Student: Yes, I did. We had the thing before school starts. He said this is not going to be a very good class. This is going to be cooperative learning. You guy's are going to earn lollipops and I was so shocked. I said this is a math class. Crazy! (laughter!!)

Interviewer: Other than lollipops, what kind of rewards did you get?

Student: Well, just the whole class claps and it just makes you feel good. Everybody has a smile. The whole class knows that you did good.

Interviewer: Can you think back in your other classes and recall how your teachers reinforced your good grades?

Student: Well, last year nothing. I didn't like my last year's class at all.

Interviewer: What were some of the differences?

Student: It was boring. You just sat there and did your work and handed it in. Did your homework at night.

Interviewer: Do you feel that team learning is a better way to learn?

Student: Yes.

Interviewer: Why?

Student: It's pretty obvious, it's just funner. More fun to learn. I kinda look forward to it. Not only because it's the last period of the day, but it kinda makes you want to learn. Instead of (Oh! so boring I hate math!) I have to admit I hate math, but now it's not that bad. He puts it in a fun way.

CHAPTER 2

A FORMAL MODEL OF SCHOOL REWARD SYSTEMS

by Suk Kang

In this chapter we present a simple mathematical model that describes how much effort students will put into preparing for an examination. The model offers a framework for answering the following questions:

- Do students work harder if it is harder for them to pass the examination?
- What types of students drop out when the passing standard is raised?
- How does a pass-fail reward system compare to a graduated reward system?
- Is it possible to induce students to work harder by introducing a handicap system? If so, how should handicaps be determined?

2.1 Pass-Fail Reward Systems

Suppose students are given a qualifying examination in which the outcome of the test is determined in the following manner: if the student's test score is above a certain critical value, he or she receives credit (passes); however, if he or she fails, no credit is given. The test score depends on the student's effort and ability and on other random factors. The randomness of the outcome lies in the fact that (1) the test is an imperfect measure of knowledge and (2) the students are unable to predict exactly how much they will learn if they put a certain amount of effort into the task. Students value the credit because it improves their future prospects and because they receive immediate praise and approval from their parents and teachers. However, they may not enjoy preparing for examinations.

Throughout this section, it is assumed that students choose their level of effort by considering the net expected benefit from passing the exam and expected effort (cost) required to pass it. Also, the students are assumed to be risk neutral, that is, their evaluation of outcomes depend on the expected value of net benefit. We assume that the test score is determined by the following relationship:

(1) $T_i = T_{io} + E_i + U_i$

where T_i is the i-th student's test score, T_{io} is the initial ability defined as the expected value of i-th students' test score if he or she doesn't put any effort into preparing for the exam, E_i is the effort level, and U_i is the random factor with variance σ_i^2. The probability of passing the exam is then written as follows:

(2) $$\begin{aligned} Pr(T_i \geq TP) &= Pr(T_{io} + E_i + U_i \geq TP) \\ &= Pr\,(T_{io} + E_i - TP \geq - U_i) \\ &= F\,(T_{io} + E_i - TP), \end{aligned}$$

where TP is the passing score and F is the cumulative density function of the random variable $-U_i$.

Denoting the benefit of passing the exam by W and the cost of effort by $C(E_i)$, the objective function of student i is written as

(3) $$\max_{E_i} W \times Pr(Pass) - C(E_i)$$

or

(4) $$\max_{E_i} W \times F(T_{io} + E_i - TP) - C(E_i)$$

The solution to the preceding optimization problem depends on the values of W, T_{io}, TP, σ_i^2 and the shape of the cost function, C. In particular, the distribution function and the cost function play crucial roles in determining the response pattern of effort to the change in critical value. In what follows we derive optimal effort level by specifying the distribution function and the cost function.

Assuming that the random factor follows the normal distribution with a mean of zero and variance of σ_i^2, the first-order condition for the optimization problem is written as:

(5) $$W \times f((T_{io} - TP + E_i)/\sigma_i)/\sigma_i - C'(E_i) = 0,$$

and the second-order condition is as follows:

(6) $W \times f'((T_{io} - TP + E_i)/\sigma_i)/\sigma_i^2 - C''(E_i) < 0,$

where f is the probability density function of the standard normal distribution.

In general, there are multiple values of E_i that satisfy the conditions for a local maximum, and the optimal level of effort is not a continuous function of TP. More specifically, when the critical value is low, students tend to work harder as teachers raise the passing standard; however when the passing standard becomes too high (the exam is too difficult), students tend to lose motiviation to work hard because they feel that the effort required to pass the exam is too high compared with what they receive for passing the exam. In the latter case, the student will decide to "take it easy" rather than to put more effort into preparing for the exam.

We show a numerical example that exhibits the pattern described above, and derive several implications from the example. Let the cost function be a constant elasticity type with increasing marginal costs:

$$C(E) = E^{\beta}, \beta > 1$$

The first-order condition is given by the following equation:

$$W \times f((E_i + T_{io} - TP)/\sigma_i)/\sigma_i - \beta E_i^{\beta-1} = MR(E_i) - MC(E_i) = 0$$

where $MR(E_i) = W \times f((E_i + T_{io} - TP)/\sigma_i)/\sigma_i$ and $MC(E_i) = \beta E_i^{\beta-1}$

Figure 1 depicts the point at which the first-order condition is satisfied for a particular value of the passing standard. In figure 1 the MR curve is drawn for $TP = T_{io}$. In this case the MR curve is the standard normal distribution function multiplied by W/σ_i and centered around zero.

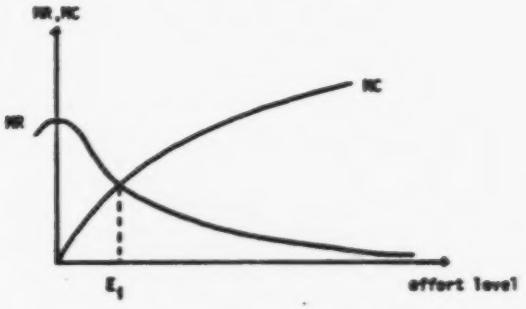


Figure 1 Determination of Effort Level

If T_{io} - TP = 0, the optimal effort level is given by Ei at which MC(Ei) = MR(E_i). As the passing standard increases the MR curve shifts to the right, and the point of intersection also moves to the right.

However, at some point, as the MR curve shifts farther to the right, it intersects the MC Curve three times. Figure 2 depicts such a case.

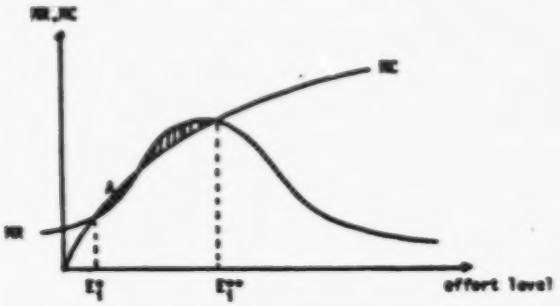


Figure 2 Determination of Effort Level

At $E_1 = E_1^*$ and $E_1 = E_1^{**}$, both the first-and the second-order conditions for the local optimum are satisfied. The optimum effort level is determined by the comparison of the areas A and B. If the area of B is larger than the area of A, the optimum effort level is E_1^{**}; however, if area A is larger than area B, the optimum effort level is E_1^*. Thus, the probability of passing the exam drastically changes when the inequality between the two areas reverses. When E_1^{**} is the optimal choice (area A < area B), the probability of passing the exam is far greater than 50 percent but when E_1^* is the optimal choice (area A > area B) the probability of passing is very low. In effect, the students have dropped out of the competition. At the point where the area of A coin-

cides with the area of B, there is a discontinuity in the reaction of student effort to an increase in the passing standard. Below the discontinuity, increases in the passing standard tend to stimulate effort. At the discontinuity, an increase in the passing standard causes effort to drop to zero. In other words, the student drops out from competition in spirit, if not in body.

Figure 3 depicts the relationship between the passing standard and the effort level. Note that as the student's effort level changes along the curve in figure 3, the probability of passing the exam also changes.

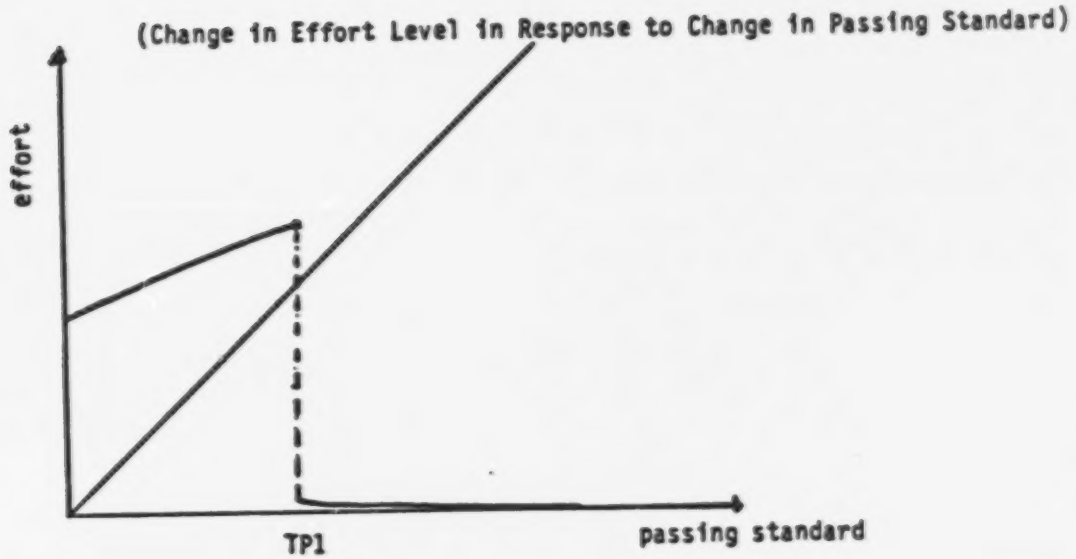


Figure 3 Reaction Function

Table 2 shows the change in the student's effort level and the corresponding probability of passing in response to an increase in the passing standard. The numerical values in table 2 are calculated by setting $\sigma_i = 0.5$, $W = 4.2$, $T_{io} = 0$, and $\beta = 1.5$. As the passing standard is raised, the student will increase his effort level. At a low level of the passing standard, the probability of passing is very high (91% for a passing standard = 0.2), however, effort level is also low. The probability of passing decreases steadily and effort level increases as the passing standard rises. However, when the passing standard becomes higher than 1.80, the student will stop trying to prepare for the exam, and the probability of passing becomes almost zero.

The i-th student's effort level is maximized when the passing standard is 1.80. Thus, if the teacher's objective is to induce students to extend their maximum effort, 1.80 is the optimal value of the passing standard.

However, setting the passing standard at 1.80 may not be a wise choice. When there is uncertainty in the student's cost function, initial ability, and/or subjective assessment of the variance in outcome, the teacher will not know the exact value that represents the optimal passing standard. If the teacher sets the passing standard too high, the student will give up and drastically reduce his or her effort. If the teacher sets the passing standard somewhat below the 1.80, the student will reduce his or her effort only slightly. Thus, in the presence of uncertainty, it is desirable to set the passing standard somewhat below the point at which the student will decide to give up (for example, 1.8 in the example in table 2). The degree to which the desirable level of passing standard is lower than the point estimate depends on the accuracy of information on T_{io} and β. The more accurate the information, the smaller the divergence.

Shifts in the Response Function

The response function in figure 3 shifts when the parmeters representing the initial ability (T_{io}), the variance in test score (σ_i^2), and the return from passing the exam (W) change. Figures 4A through 4C depict shift patterns of the reaction function when T_{io} is increased (4A), when σ_i^2 is decreased (4B) and when W is increased (4-C). In the figures, the curve 1-1-1-1 represents the reaction of student 1 that corresponds to figure 3.

TABLE 2

EFFORT LEVEL AND PROBABILITY OF PASSING IN
RESPONSE TO CHANGE IN PASSING STANDARD

$\sigma = 0.5$, $W = 4.2$, $T_{io} = 0.0$, and $\beta = 1.5$.

Passing Standard (Required test score)	Effort Level (expected test score)	Probability (of passing)	Expected Reward
0.20	0.87	.910	3.821
0.40	1.03	.896	3.764
0.60	1.20	.885	3.717
0.80	1.37	.873	3.666
1.00	1.55	.864	3.630
1.20	1.72	.851	3.573
1.40	1.90	.841	3.534
1.60	2.07	.826	3.471
1.80	2.25	.816	3.427
2.00	0.01	.000	0.000
2.20	0.01	.000	0.000
2.40	0.01	.000	0.000
2.60	0.01	.000	0.000
2.80	0.01	.000	0.000
3.00	0.01	.000	0.000

The T-T-T-T curve in figure 4A is the reaction function of the person whose initial ability is higher than that of student 1. As the figure shows, those with higher initial ability put forth less effort for a given passing standard. On the other hand, they are also less likely to withdraw from the competition.

The effect of a change in the perceived importance of random factors in determining performance on the test is depicted in figure 4B. The S-S-S-S curve represents a student who believes σ_ϵ^2 to be small (that is, who believes that the test is reliable). The 1-1-1-1 curve represents a student who believes σ_ϵ^2 to be large. When it is perceived that passing the test is determined by chance rather than effort (that is, σ_ϵ^2 is large), the payoff for effort goes down even though the impact of effort on learning has not changed. Students are more likely to drop out of the competition, and those who remain in the competition will typically put less effort into it. Only when the passing standard is very low will an increase in the randomness of the test stimulate effort. The model implies that effort can be enhanced and dropouts forestalled by using tests that are more reliable and that are more responsive to effort and by enhancing the student's sense of self-efficacy (that is, the belief that effort rather than luck determines outcomes).

Another factor that influences the student's reaction is the magnitude of the reward (W). When the reward for passing the examination is large, the students will put forth more effort. Figure 4C shows such a relationship. The curve W-W-W-W corresponds to the student W who expects a the larger reward for passing the exam than is student 1. The increased reward reduces the probability that students will drop out of the competition and, at all levels of the passing standard, student W puts forth more effort than student 1.

In summary, students increase their effort when rewards for passing a test are raised and more reliable tests are used. Reductions in the cost of learning and effort (because of improved teaching or parental encouragement) also increase effort. Raising the passing standard increases effort up to a point. Beyond that point, the student stops trying, and drops out of the competition because the effort required to pass the test is more costly that the expected reward.

Figure 4-A Change in Reaction Due to Difference in Initial Ability

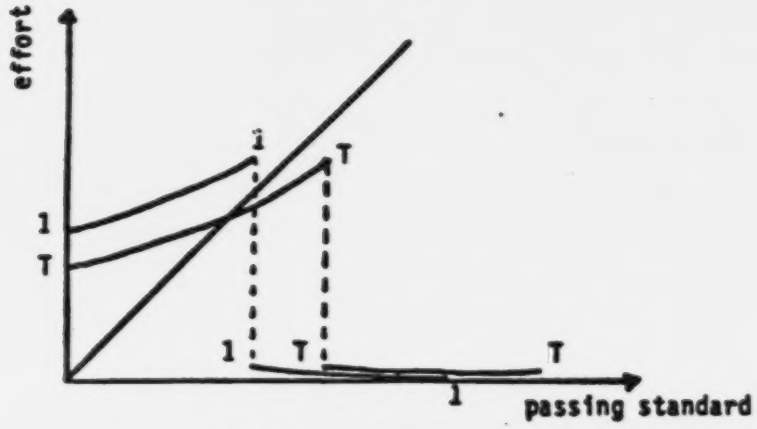


Figure 4-B Change in Reaction Due to Difference in the Variance of Test Score

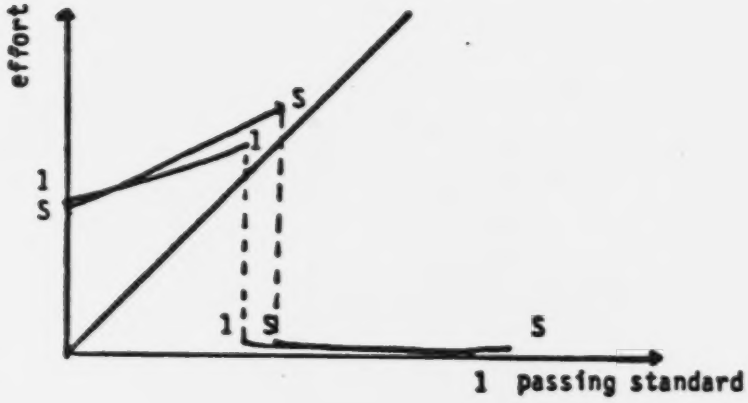
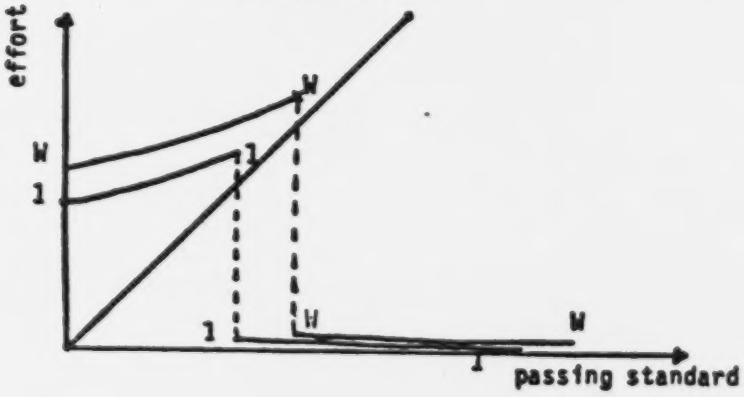


Figure 4-C Change in Reaction Due to Difference in the Reward



2.3 Graduated Rewards

An alternative approach to rewarding students is that of distributing graduated rewards for improvements in performance. In the graduated reward system the reward is determined by $P \times (T_i - T_{io})$, where P is the reward per point on the test and T_i is the i-th student's test score. The student will choose his or her effort by equating the marginal cost of effort to the marginal return from additional effort. The objective function is written as

$$\max_{E_i} [P \times (E_i + U_i) - E_i^{\beta}].$$

The optimal effort level is given by

$$E_i = (P/\beta)^{1/(\beta-1)}$$

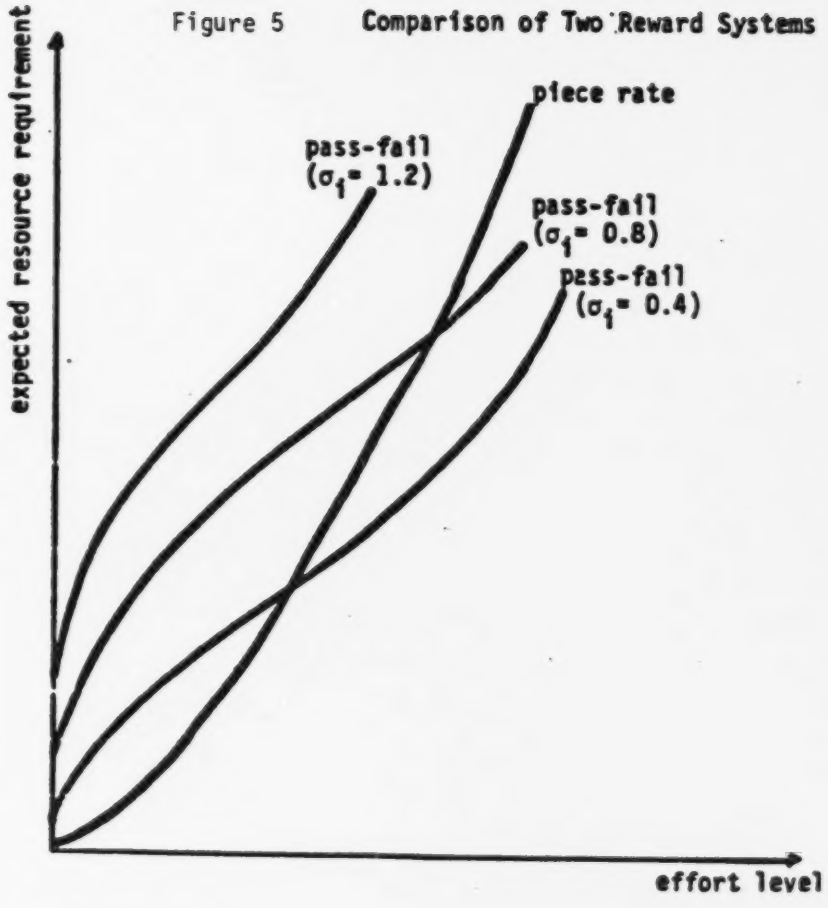
and the expected value of payment is

$$P \times [(P/\beta)^{1/(\beta-1)}].$$

How well does a graduated reward system stack up against a pass-fail reward system? The efficiency of the two systems will be evaluated by comparing the expected reward required to induce students to put forth a certain effort level in each system. If one system induces more effort than the other at given levels of expected reward, it is more efficient. The expected cost in the pass-fail system is obtained by multipling W by the probability of passing. The expected cost in the graduated reward system is obtained by multiplying the expected value of the test score gain $(T_i - T_{io})$ by the reward per test point (P).

The cost efficiency of the pass-fail system is obtained in the following manner: Given the error variance of the test, the optimal strategy in the pass-fail system is to choose both the size of the reward and the level of passing standard so that effort is maximized at each level of expected expenditure. By changing the size of the reward and tracing the passing standard that maximizes the student's effort level, the corresponding expected resource requirement is obtained. The efficiency of the two systems is compared for each level of the expected resource requirement. Figure 5 shows the relationships between effort and resource requirement in the two systems. For the pass-fail system, the curves are drawn for three levels of the error variance of the test.

Figure 5 Comparison of Two Reward Systems



The motivational impact of a pass-fail test is greatest when all students start with the same level of knowledge and skill and learn at the same rate. Criterion-referenced tests of new material more closely approximate this situation than do competency tests for graduation. Since, however, background knowledge is often essential to learning new material and learning progresses at different rates in different people, there is no way out of the dilemma when tests are graded on a pass-fail basis. When one sets the passing standard, some students will find the standard so easy to achieve that they will not put out more than minimum effort (if that is all that motivates them) and other students will find is so hard they will probably give up trying.

CHAPTER 3

THE DESIGN OF HANDICAPPING SYSTEMS FOR REWARDING LEARNING: A MATHEMATICAL ANALYSIS OF INTERTEMPORAL ISSUES

by John Bishop and Charles Wilson

The primary objective of school recognition of learning is to induce students to push themselves closer to their potential. Schools cannot, however, reward everybody for everything. There is a limit to how much recognition a school can hand out, and handing out too many awards may reduce the perceived value of the award. If recognition goes only to those who score at the top of an absolute scale of knowledge and skill, most students will have no realistic chance of receiving significant rewards. This often results in these students' giving up on the task of learning and disrupting the learning efforts of others. A reward system that gives everyone a chance at recognition and rewards only the learning that occurs because of extra effort rather than natural ability is likely to be a more effective stimulator of learning than is general recognition of all learning. Since effort cannot be measured, the system cannot reward for effort. Learning can be measured, but what is needed is a handicapping system that gives the slower students a more equal chance of being recognized for academic achievement. Unfortunately, there are difficulties in finding an administratively feasible way of measuring how much a student would learn in the absence of the reward system. The key to constructing an effective system of recognizing academic achievement, therefore, hinges on finding an efficient way of defining a base score above which recognition is given.

3.1 Alternative Reward Systems

We will study systems of recognizing student learning with three contrasting ways of defining the base score of achievement above which graduated rewards are offered: a fixed base score, a monthly updated base score reflecting previous month's achievement, and a base score computed on the student's peak performance over all previous months.

The simplest program fixes a base score in period 0 at some level, L, and lets it grow (or decline) in each period by a constant factor, q. Although such a policy may be effective on a temporary basis, it is not necessarily desirable over a period of many months. A student's performance on a test at

the beginning of a school year is probably a reasonable predictor of performance during the first semester. However, it may be a poor predictor of his or her performance at the end of the school year or in the following year.

The fixed base score policy will serve as a useful benchmark for evaluating the efficiency of various alternative programs. The primary virtue of such a recognition system is that the actions of individual students in one period do not affect the recognition that the student can collect in the following periods.

One commonly proposed recognition system is that of rewarding progress or improvements over the student's performance on a previous test. The specific system that will be evaluated offers graduated rewards for raising one's score on a test above one's performance on the previous test. Declines in the score are not penalized. One might also wish to incorporate a growth or depreciation factor into the update rule, so that the threshold in each period is merely some multiple of last period's test score. In this case, the base score in each period t, $\bar{L}_t$, is defined by $\bar{L}_t = pL_{t-1}$, where p is some positive constant, and L_{t-1} is test score in the previous period. This type of recognition tends to be inefficient. Since a reduction in the test score on one test lowers the base score in the next, students may have an incentive to adopt a two-period cycling strategy in which they alternatively increase and then decrease their effort level, thereby collecting the reward every other test. This tends to generate a relatively large reward payment, but very little net increase in learning.

An alternative method of updating the base score that eliminates any incentive to decrease learning is that of using the student's highest test score since the beginning of the school year. A slight generalization of this idea is to define the baseline by $\bar{L}_t = q \max(L_{t-1}, \bar{L}_{t-1})$, where q is a positive constant. If test score exceeds the base score in any period, the base score is automatically raised for all future tests. If test scores stay below the current base score, then the future base scores are unaffected. In this case, it simply increases (or declines) by a factor of q. Setting $q = 1$ yields a pure peak system of recognition. Real-life examples of this reward system are found in track and field and in swimming where improvements in one's best time are an important motivator for the athlete.

3.2 The Model of Behavior

We use a standard intertemporal model of individual behavior to analyze the response of students to the various types of reward systems. The student is assumed to choose an effort level in each period that maximizes the discounted sum of utility (rewards minus costs of effort) over an infinite planning horizon.[1] The cost of effort in any period is a quadratic function of learning with no costs of adjustment. The assumption of no costs of adjustment and no interdependence of the learning and cost functions over time means that how much a student can learn in the nth period does not depend on how much he or she learned in previous periods.[2] Another crucial assumption is that, from the student perspective, the relationship between effort and test score does not have any random elements--for example, the test measures learning without error and the student can correctly predict how effort will translate into a test score (this assumption is relaxed in section 3.6). We also assume that, in the absence of any reward system, the student's test score will increase or decline in each period by a constant factor g, which is perfectly anticipated. A formal development of the model is presented in appendix 3-A.

The purpose of the model is to calculate the partial equilibrium responses of the student to the introduction of a permanent school reward system which rewards achievement over an individualized baseline. Therefore, we assume that other aspects of the environment such as the quality of teaching are unaffected by the reward system.

3.3 Measures of Cost-Effectiveness

Besides analyzing their impact on the student's behavior, we will also compare the effectiveness of these three alternative systems. As one might expect, the marginal costs of improving one's test score increase with the size

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[1] Infinite planning horizons are assumed in order to examine the properities of handicapping systems when the system is in place for many years in a row. This creates a real challenge for a handicapping system because the student's individualized goal must carry over from 1 year to the next.

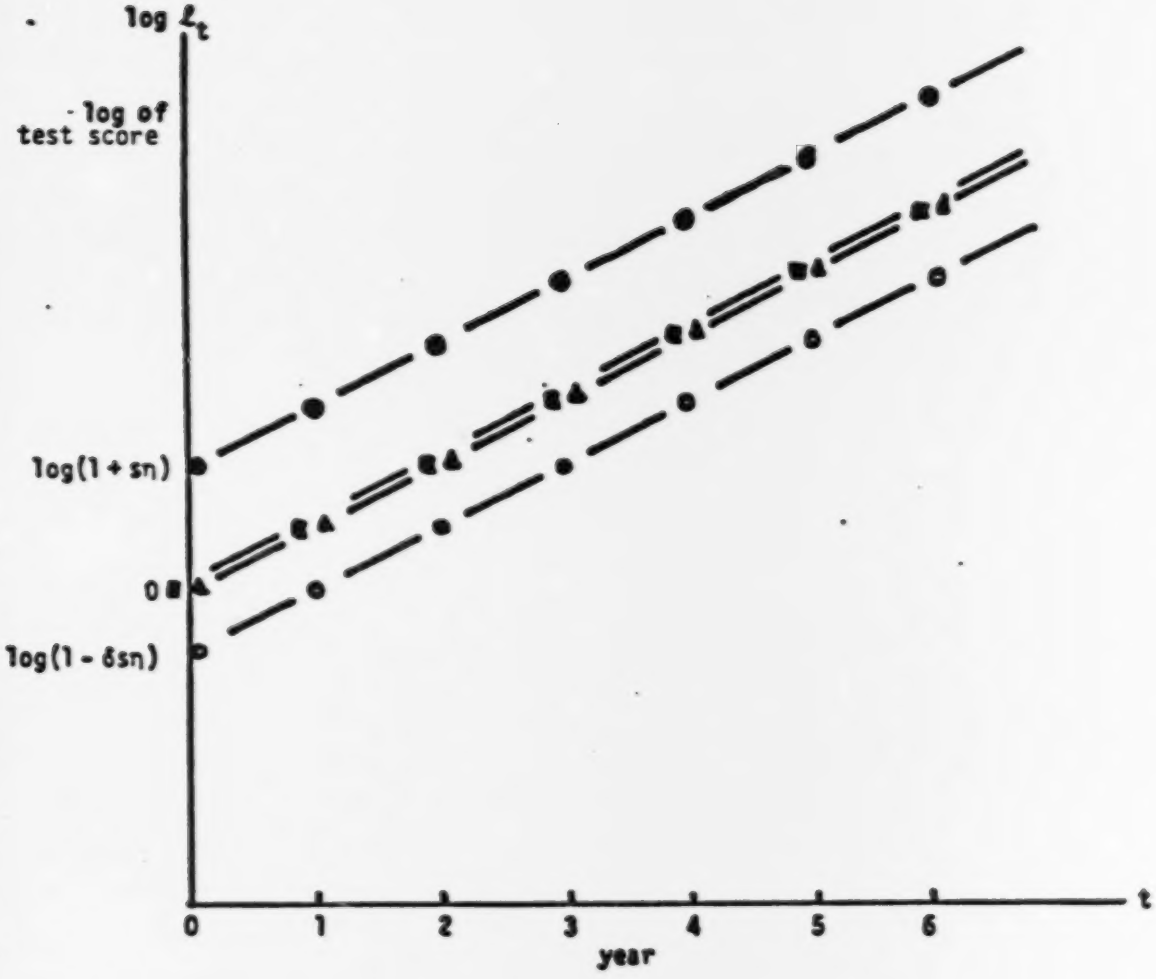
[2] Learning independence across time periods might occur because (1) cummulative learning is occurring at the student's own pace or (2) the pace at which material is presented is fixed, learning new material does not depend on knowing old material and what is at issue is the degree of comprehension of the material presented.

of the improvement that is desired. For a given reward structure, larger increases in learning will require more than proportional increases in rewards. Consequently, when comparisons across reward structures are made, rates of reward will be adjusted to produce equal increases in learning. The effectiveness of the different reward systems for specific students also depends on the rate at which the student would have learned in the absence of the reward system. Therefore, comparisons between programs will be made while holding the students underlying learning rate constant. The indicators of cost-effectiveness will be the cost to the school of the rewards offered divided by the extra learning that is induced. Since different reward systems generate different flows of learning and reward costs over time, a discount rate is essential to evaluate learning gains over time. The discount rate, δ, which the student uses to discount future reward reflects the rate at which the student values future rewards relative to current rewards. If this also corresponds to the social rate of discount, then δ is the appropriate factor for discounting both the subsidy and distortion costs. We will begin by assuming that society discounts future learning at the same high rate as does the student. In section 3.6, we will examine the effect of assuming that students are much more present oriented than society.

3.4 How Students Respond: A Comparison

Before turning to an evaluation of the relative effectiveness of the three reward systems described in section 3.1 it will be useful to explore in some detail the differences in how students respond to these systems. Besides the rate at which learning is rewarded, the major determinant of the student's response to a reward system is the growth rate of the student's learning ability relative to the rate at which the base score depreciates. It is important, therefore, that we compare the student's response to different reward systems for various exogenous growth rates of knowledge. To simplify the exposition, we will assume throughout that the base score depreciation rate for each system is zero. In this case, the updating rule for (1) the fixed base score system becomes $\bar{L}_t = \bar{L}_{t-1}$; (2) the updated base score system, $\bar{L}_t = L_{t-1}$; and (3) the peak base score system, $\bar{L}_t = \max(L_{t-1}, \bar{L}_{t-1})$.

A student's response to the different systems when his or her learning ability is growing extremely rapidly is illustrated in figure 6. In each case,



○ learning with no reward system

● learning under fixed baseline reward system: $\bar{L}_t = \bar{L}_{t-1}$

▲ learning under updated baseline reward system: $\bar{L}_t = L_{t-1}$

■ learning under peak score baseline system: $\bar{L}_t = \max(L_{t-1}, \bar{L}_{t-1})$

Figure 6. Student response to different reward systems.

the student will respond by increasing learning by a fixed percentage in each period. The only difference in the systems is the magnitude of the response. Under a fixed base score system, the student's actual test scores have no effect on the future base scores. Consequently, learning is increased by a factor of $(1 + sn)$ where s is the proportionate increase in marginal rewards for learning resulting from the reward system and n is the elasticity of learning with respect to total rewards. Under the other two programs, however, increasing learning in any period also raises the base score for the next period. Therefore, the student will choose an effort level which equates the marginal gain from increasing learning today to the marginal loss from raising the base score in the next period. As a result, learning is increased by a factor of only $(1 + (1 - \delta)sn)$, where δ is the discount factor for future costs and rewards. This means that, for a discount factor of .9 per month, a 10-fold increase in the reward rate is required to generate the same learning response under the monthly updated and peak period system as under the fixed base score programs.

Inspection of the formulas in appendix 3-B reveals that as long as students follow a steady response to each reward system and the subsidy rates are adjusted to stimulate the same increase in learning, then the reward costs of each program are identical. Although monthly updated and peak period systems require a higher rate of reward, the base score is continually updated so that a much smaller fraction of the test score is actually rewarded. Unfortunately, as the reward rate is increased, students require a higher growth rate before they will respond with the steady response illustrated in figure 6. For a reward rate of .4 and a discount factor of .9 per month, students with monthly updated base scores must have an exogenous learning growth rate in excess of 9 percent if they are to adopt a steady response strategy. Learning ability clearly cannot grow at such a rate for long.

For a range of lower growth rates, students facing the fixed or the peak score baselines will continue to adopt a steady response strategy. However, students facing the monthly updated baseline system will adopt a learning strategy in which the reward is collected once every two periods. In the first period, they will increase their learning by a factor of $1 + s^n$; in the next period, they will decrease it by a factor of $1 - \delta s^n$. Then the cycle starts again. The result is illustrated in figure 7. Note that in those periods when

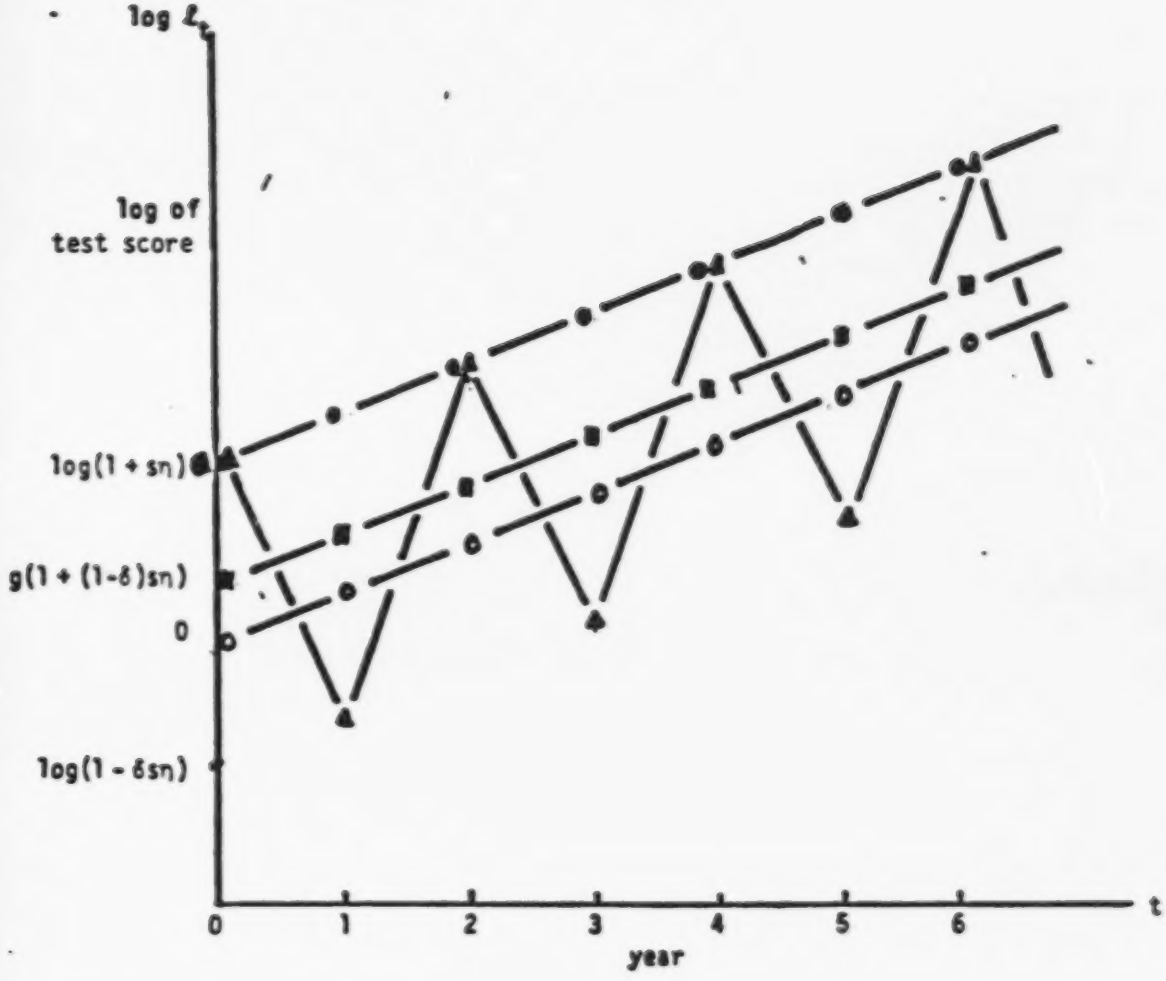


Figure 7. Student response to a monthly updated baseline system.

the reward is actually collected, the student responds exactly as if the baseline were fixed. This reflects the fact that since the student does not plan to exceed the baseline in the next period anyway, the learning today cannot affect future rewards. In those periods when the threshold is not exceeded, however, the student does plan to collect the reward in the following period. Consequently, the student decreases his learning below the presubsidy optimum in order to lower the next period threshold. As a result, the gain in learning in the expanding periods is at least partially offset by the learning decline in the contracting periods. As we might expect, the net result is a relatively large increase in both the distortion and reward costs.

For growth rates near zero, the learning of students facing a peak score baseline will also begin to cycle, as illustrated in figure 8. After the student initially increases his or her learning to collect the reward, the threshold in the following period may be sufficiently high that the student chooses to remain at his or her prereward system learning effort. As the ability of the student continues to grow, however, he or she will eventually choose to raise learning beyond the threshold established in the initial period and collect the subsidy again, thus starting a new cycle. If the students plan to wait n periods before collecting the subsidy again, the optimal learning in every n-th period is increased by a factor of $(1 - \delta^n)s^n + 1$. In all other periods, it remains unchanged. The length of the cycle increases as the growth rate of the student's ability decreases and it goes to infinity as the growth rate goes to zero. For most parameters, however, the range of growth rates for which any cycling occurs is relatively small. For a subsidy rate of .4 and discount factor of .9 per month, cycling occurs only for growth rates between 0 and 1 percent. In figure 8, a three period cycle is illustrated.

Note that even in the periods when the student collects a reward, the response is less than it would be under a fixed baseline reward system. This is because any increase in learning today that exceeds the baseline imposes a cost in the form of a higher baseline in the future. However, as we should expect, the longer the student plans to wait to collect the reward again, the lesser the cost and, hence, the greater the learning response. The important point to emphasize is that, in contrast to a monthly updated baseline system, the student never has an incentive to reduce learning below the prereward system level.

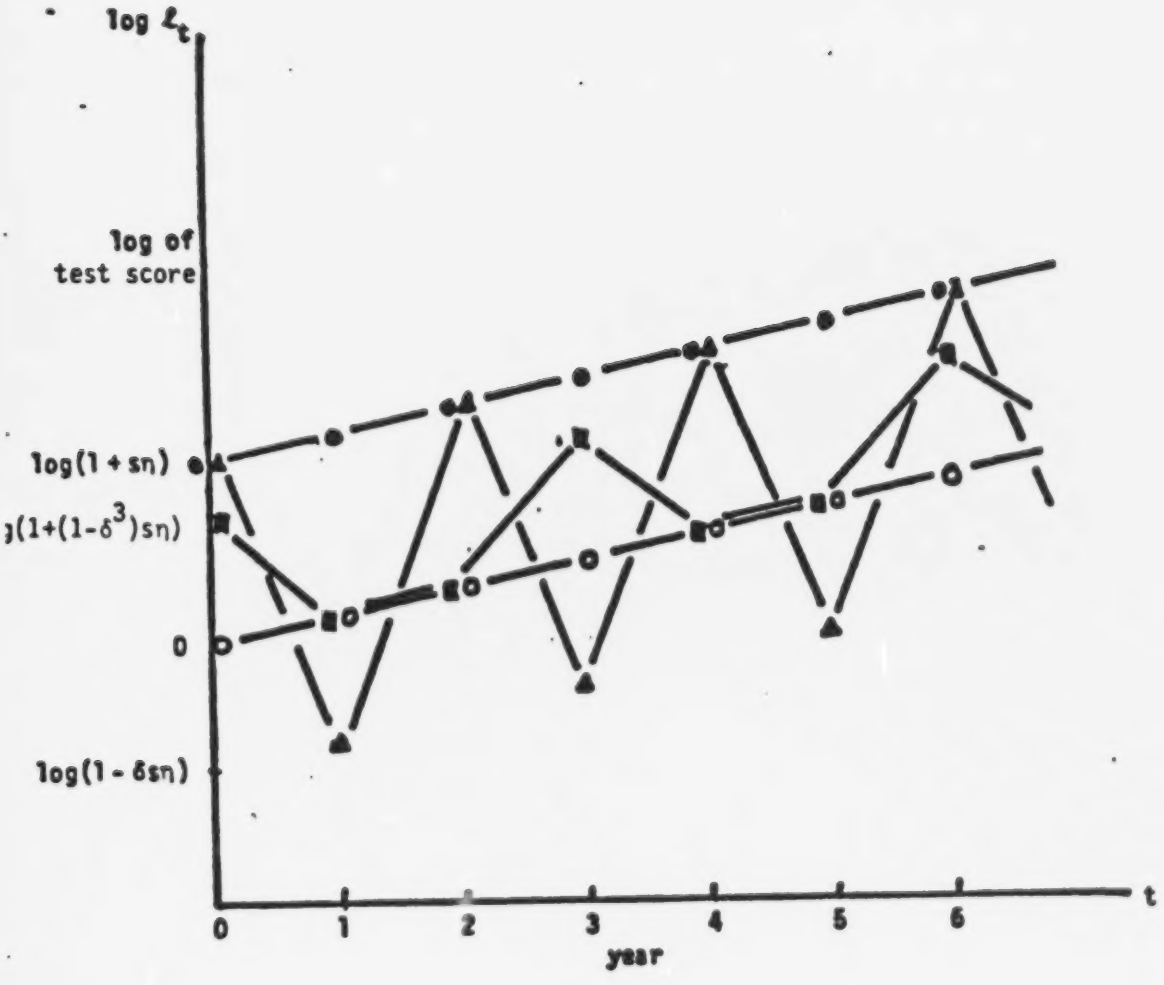


Figure 8. Response of students facing a peak score baseline system.

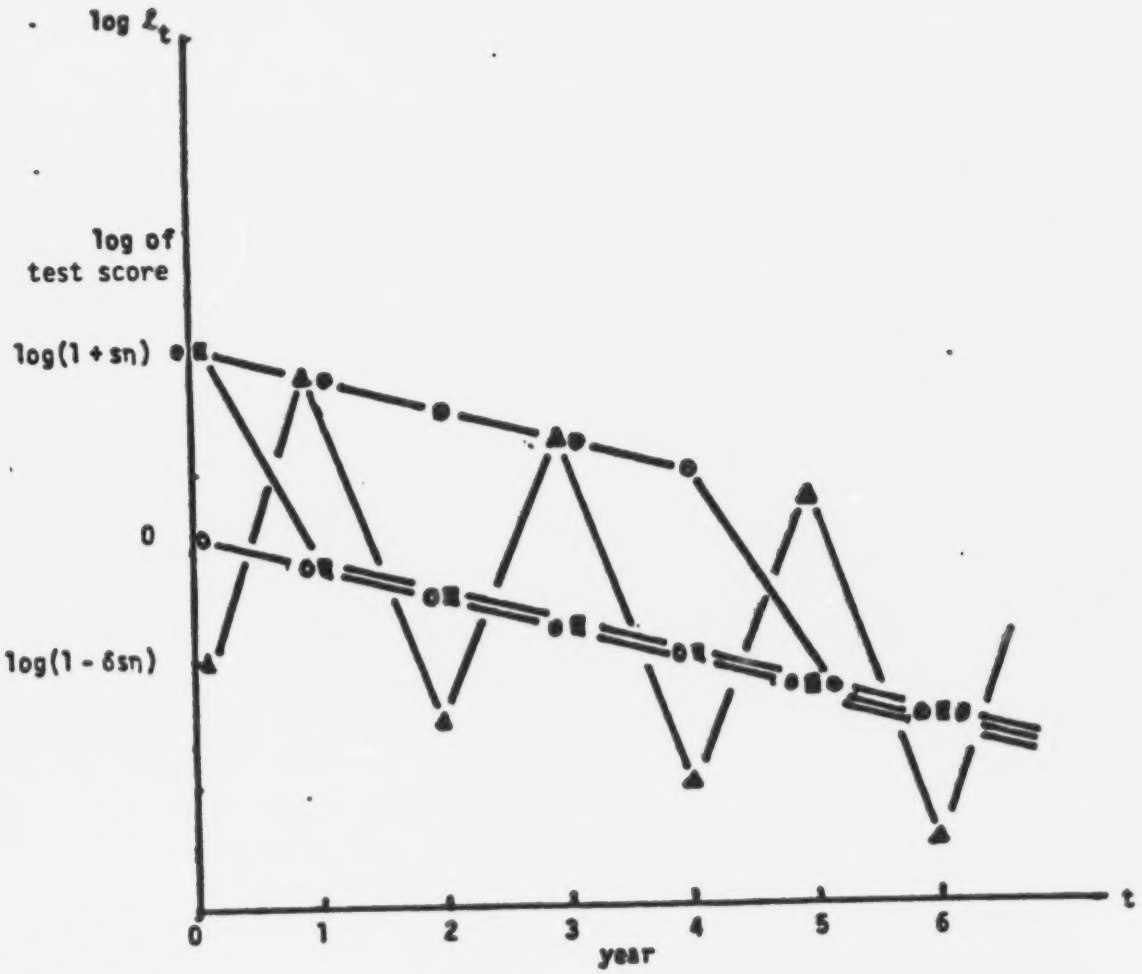


Figure 9. Typical student learning responses under different reward systems.

If the expected growth rate of the student's test score is negative, students facing peak score baselines will collect the subsidy once at most. Students facing a fixed baseline may choose to collect the reward for one or more periods, but eventually they too will stop responding to the subsidy. Except for extremely large negative growth rates, the learning of students facing the monthly updated baseline will continue to exhibit the two period cycle, but if the growth is sufficiently negative, they may choose to begin the cycle by first decreasing learning. Since later learning is discounted, it turns out that whenever this strategy is adopted, the present value of the learning gain is negative. For a subsidy rate of .4 and a discount factor of .9, this is the result whenever the growth rate lies between -1 and -9 percent per months. Typical learning responses under the different reward systems for growth rates in this range are illustrated in figure 9.

Except for the fixed baseline reward system, the most efficient responses to these reward systems tend to occur at the higher growth rates. The effect of introducing a depreciation factor into the threshold updating rules is essentially equivalent to an increase in the growth rate of the student's learning ability. In the next section, we will examine the cost effectiveness of these systems with and without a baseline depreciation factor.

3.5 Cost-Effectiveness of Different Reward Systems

It is important to recognize that the cost per unit of stimulated learning will generally be sensitive to the magnitude of the learning increase. The greater the increase in learning, the greater is the cost per unit of learning. In order to obtain a more appropriate comparison of the different programs, therefore, we will adjust the rate of reward for each program so that presently discounted learning rises by 1 percent at growth rates for which firms respond with a steady increase in employment (see figure 6). In general, this will imply different rates of reward for different programs.[1]

[3] The relative cost-effectiveness of the different reward systems may be sensitive to the level at which the baseline is set at the beginning of the program. For instance, if the initial baseline is set very low, the fixed baseline program is essentially reduced to a nonmarginal reward of learning. Clearly, raising the baseline in this case will result in a dramatic reduction in the reward costs. For the other systems, an increase in the initial baseline will also lower the reward costs; however, the decrease will not be as significant. If the initial threshold is set too high, however, students may not respond at all or, in some cases may even respond by initially reducing

Fixed baseline: $\bar{L}_t = q\bar{L}_{t-1}$. As long as the growth rate of the baseline does not exceed that of the student's ability, a student will respond to a fixed baseline reward system exactly as he or she would to a nonmarginal reward system. For a reward rate of s, the student will increase learning in each period by a factor of (1 + sn), where n is the elasticity of demand for learning. The distortion costs are also the same. The major difference in the two programs is in the costs of the reward system.

Let us define the reward learning ratio of any program as the reward cost divided by the increase in the students learning. Then, using the formulas reported in appendix 3-B, we find that a nonmarginal reward system generates a reward-learning ratio of $(s + \frac{1}{n})$ whereas a fixed baseline system that grows at the same rate as the student's test performance generates a reward learning ratio of s. This means that for an elasticity of learning demand equal to .25 and a reward cost of .04, the reward costs generated by a nonmarginal reward system are at least 4 times the amount by which the learning increases, while the reward-learning ratio for the fixed baseline system is only .04. In both cases, learning is increased by 1 percent. If student test-taking ability grows faster than the baseline, the fixed baseline system becomes less efficient. For a constant threshold, the learning ratio rises to .43 if student test-taking ability grows at 1 percent, 1.94 if it grows at 5 percent, and 3.67 if it grows at 10 percent. Other comparisons are presented in table 3. The fixed baseline system is almost always more efficient than a non-marginal subsidy. The problem with the fixed baseline system is that the individual rather quickly stops responding to the incentives of the reward system if test-taking ability is declining.

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their learning. It is important, therefore, that our calculations be based upon the appropriate initial baseline. Last periods test score is a reasonably good predictor of this periods test score, so we will assume that schools would base the period 0 baseline for each student on their test score in period -1. In implementing our updating formulas, therefore, we will equate the baseline in period 0 with the presubsidy optimal learning for that period.

TABLE 3

REWARD-LEARNING RATIOS*
(For g = 10%, learning increases by 1%)

Exogenous Growth Rates (g)	No. Baseline ($\Gamma_{t-1} = 0$)	Fixed Baseline ($\Gamma_t = q\Gamma_{t-1}$)		Immediately Updated Baseline ($\Gamma_t = pL_{t-1}$)			Peak Score Baseline ($\Gamma_t = q \max(L_{t-1}, \Gamma_{t-1})$			
							Prob of cont. π=100%		Prob of cont. π=.75	
		q=1.0	q=.95	p=1.0	p=.95	p=.90	q=1	q=.95	q=1	q=.95
10%	4.04	3.67	3.80	3.67	3.80	3.88	3.67	3.80	1.13	1.54
5%	4.04	1.94	2.67	5.89[c]	2.67	3.05	1.94	2.67	.60	1.07
1%	4.04	.44	1.68	3.89[c]	3.05[c]	2.37	.44	1.68	.13	.68
.5%	4.04	.24	1.55	3.68[c]	2.91[c]	2.25	.40[a]	1.55	.18[b]	.63
0%	4.04	.04	1.42	3.48[c]	2.77[c]	2.15	.40*	1.42	.40*	.57
-1%	4.04	--	1.15	neg[c]	2.51[c]	1.96	.40*	1.15	.40*	.46
-5%	4.04	--	.04	neg[c]	1.57[c]	1.15	--	.40*	--	.40*
-10%	4.04	--	--	--	neg[c]	.87[c]	--	--	--	--
Subsidy Rate	.04	.04	.04	.40	.276	.211	.40	.276	.123	.111

*These results are based on a discount factor δ = .9 and elasticity of learning demand η = .25.

--No response.

[a]1 period cycle.

[b]4 period cycle.

[c]2 period cycle

neg present value of learning increase is negative.

Updated baselines: $L_t = p\,L_{t-1}$. We turn now to an evaluation of reward systems which use test performance in the previous periods to establish a baseline. As we illustrated in section 3.3, student response depends on the growth rate of exogenous learning relative to the baseline depreciation factor. Consider first the case where the baseline depreciation factor, p, is equal to 1. Assuming a discount factor of .9, and elasticity of learning demand equal to .25, and a 40 percent reward rate, a students ability to perform on the test would have to grow by more than 9.8 percent per month for the firm to steady response strategy. This is very improbable.

Students with growth rates between -9 percent and 9.8 percent will respond with a two-period cycle. If the growth rate exceeds -1 percent the student will begin by increasing learning. In this case, the net increase in the present discounted value of learning will be positive but generally smaller than 1 percent. In spite of this, the reward-learning ratio rises dramatically relative to the fixed baseline system. When the student test taking ability does not change, the reward learning ratio is 3.48. Inspection of tables 1 and 2 reveal similar results for other growth rates. For growth rates between -9 percent and -1 percent, the firm will also adopt a cycling strategy; however, in these cases, the cycle will begin by decreasing learning. As we noted in section 3.3, this leads to a reduction in present discounted learning.

The performance of the updated baseline system can be improved by setting the baseline equal to a fraction of last period's test score. For instance, by setting p = .95, students whose ability is growing at a rate of 4.1 percent or higher will adopt a steady response. When a steady response is generated, costs and distortion costs become comparable to the fixed baseline system. For lower baseline depreciation factors, even more students adopt the steady response strategy, but the reward-learning ratio begins to rise as a greater fraction if the students' learning is rewarded. Some comparisons for a threshold depreciation factor of .9 are also presented in table 3.

Peak score baselines: $\bar{L}_t = q \max(L_{t-1}, \bar{L}_{t-1})$. For a given baseline depreciation factor, the cost-effectiveness of a reward system can be improved by using the peak scores on all previous tests as the baseline. In this case, the baseline updating rule can be written $L_t = \max(\bar{L}_{t-1}, L_{t-1})$. The gain from using the peak score updating rule over the immediate updating rule comes from

two sources: (1) many more students respond with a steady increase in learning and (2) the response of those students who adopt a cycling response is more stable. Under the peak score baseline system, students never have an incentive to reduce learning below the level that is optimal in the absence of a reward system.

Assume, as before, a discount factor of .9, an elasticity of learning demand equal to .25, and a 40 percent reward rate. Then, students whose test scores grow by more than 1 percent per period will adopt a steady response strategy, increasing learning by 1 percent in each period.

Students with growth rates between 0 percent and 1 percent per period will adopt the cycling strategy illustrated in figure 8. For instance, at a g of one-half percent per period, they will increase learning once every 11 periods, generating a .98 percent increase in present discounted learning with a reward learning ratio of .40. Both of these numbers are higher than they would be with fixed baseline, but they are also considerably lower than under the updated baseline.

One of the drawbacks of a pure peak score baseline is that only students with positive growth rates of learning ability will respond after the first period. One way to reach more students is to let the baseline depreciate each period by a constant factor q. This has the additional benefit of inducing more students to adopt the steady response strategy and also of generating greater learning from each student. For instance, if q = .95, the reward rate required to induce students following the steady response strategy to increase learning by 1 percent is lowered to .28. Furthermore, the steady response will now be adopted by students whose learning capacity is not declining by more than 4 percent per year. Unfortunately, lowering the baseline depreciation factor tends to increase the reward-learning ratio. In fact, the percentage increase is quite substantial for students with growth rates near 0 percent.

Although this program is substantially more cost-effective than the updated baseline system, it is still the case that relatively large reward rates are required to produce even a 1 percent increase in learning. This suggests that no feasible permanent program is likely to have much impact. However, as we shall see in the next section, when students are uncertain about the duration of the reward system or students have much higher discount rates than

society, learning responses are larger and marginal reward systems are more effective.

3.6 Uncertainty

Thus far, the analysis has been concerned exclusively with the optimal learning response of students to various reward systems under the assumption that students do not face uncertainty and have the same discount factor as society as a whole. There are at least two ways in which uncertainty might be incorporated to the analysis. First, students may be uncertain about how long the reward system will remain in place or, equivalently may have a strong preference for immediate rather than future rewards. This will not affect the behavior of the student under fixed baseline systems, but it dramatically improves the cost effectiveness of reward systems that use updating. The second kind of uncertainty that we might wish to examine is the student's uncertainty about his or her future learning function, or alternatively, the optimal amount of learning. Although we believe that this kind of uncertainty is important, a detailed analysis of behavior under these circumstances is beyond the scope of this chapter. We will confine ourselves, therefore, to a few brief remarks.

High discount rates and uncertainty about the life of the reward system. If students have higher discount factors than society, the fact that getting a reward now will raise future baselines and make future rewards less probable becomes less of a concern. In addition, under almost any circumstances in which a reward system might be introduced, there is likely to be a significant probability that it will be discontinued at some point in the near future. Perhaps the simplest way to model this type of uncertainty is to assume that there is a fixed probability, $(1 - \pi)$, that the reward system will be discontinued in any period--given that it was still in place in the previous period and that the student learns of the program's discontinuance at exactly the time at which it is discontinued. In terms of the student's objective function, an increase in this probability lowers the discount factor that the student uses to evaluate future rewards, while leaving society's discount factor unchanged.

This does not affect the behavior of students facing a fixed baseline because their current response does not depend on rewards expected in future periods. When there is updating, however, the student responds by trying harder.

This reflects the fact that future rewards are now less probable or valued less.

In evaluating the learning gains and reward system costs, there is a difference between lowering the society's discount factor and increasing the students subjective probability that the reward system will be terminated. If the school actually intends for the program to be permanent, then the social gains and costs of the program should be evaluated at a discount rate which does not incorporate the possibility that the reward system will end. The net effect of an increase in either the student's subjective probability that the system will be discontinued or his or her discount rate, therefore, is to generate more learning at given rates of reward. These gains can be quite significant.

Suppose a pure peak score baseline reward system ($q = 1$) is in place but that students believe that there is a 25 percent chance that the system will be terminated next period. Again assuming the discount factor is .9 and the elasticity of learning demand is .25, a subsidy rate of only 12 percent is required to induce students adopting the steady response strategy to increase learning by 1 percent. This is substantially smaller than the 40 percent rate required under the pure peak score update system with no uncertainty, but it is still three times higher than the 4 percent subsidy rate required under the fixed baseline system. The growth rates for which students adopt a cycling learning response still lie between 0 percent and 1 percent, and the distortion costs are also unchanged. The reward costs, however, are substantially lower. The reward learning ratio falls from .44 to .13 for students whose test performance grows at 1 percent per period.

If a depreciation factor of .95 is introduced into the baseline updating rule, the cost-effectiveness of the system for improving students is reduced; however, it still compares reasonably well with the fixed baseline system, even without a depreciation factor. Except for growth rates near 0 percent, the reward-learning ratio is still lower under the peak score updating system, and more students respond to the system. A systematic comparison of the cost effectiveness of the peak updating under uncertainty with the other programs is presented in tables 2 and 3. These results suggest that peak score updating may be considerably more effective if students are even a little uncertain about the permanence of the reward system.

Uncertainty about future test scores. The basic principles that govern the student's response to a reward system under perfect certainty do not change when we introduce uncertainty about his or her future optimal learning. It remains the case that the student's response under a fixed threshold program depends only on the relation between increasing learning today and raising the baseline in the future. The difference is that the size of the future baseline relative to its future optimal learning level is now a random variable.

Although there may, consequently, be more fluctuations in the learning response in this case than there would be under perfect certainty, we should still expect peak score updating systems to be more cost-effective than immediate updating. The reason for this is the same as in the previous case. Immediately updated baselines sometimes produce an incentive to lower present learning below the optimal level in order to lower the baseline in the next period. Under a peak score baseline, the future baseline can only be affected if the baseline is exceeded in the current period. Since future rewards are discounted, the student will never choose to lower learning below the optimal level in the current period. We see no reason, therefore, why the presence of uncertainty should change our conclusion that a peak score baseline is a more cost-effective way of increasing learning than is immediate updating.

Baselines that are averages of past scores. Updating individualized learning expectations by taking averages of past scores seems to be an attractive way of dealing with uncertainty regarding learning rates and the fact that monthly tests measure true learning with some error. Such reward schemes were not subjected to a rigorous mathematical analysis. However, our examination of the simpler case of immediate updating leads us to conclude that updating baselines with a running average of past performance has many of the same problems, though generally to a lesser degree. The student response to the subsidy is still reduced by the fact that future baselines will be higher and rewards harder to get. The greater average delay in the response of baselines to past performance means that the behavioral responses are greater for a given rate of subsidy. When there is no uncertainty about the effect of effort on learning rewards, updating based on a running average will reduce cycling. Test performance has a random component, however, and this, combined with the fact that the student can purposely do poorly on the test, reopens the possibility of

cycling. Clearly, a lower bound must be placed on the scores that are included in the average which updates the threshold.

3.7 Conclusion

What may be concluded from the exercises we have reported in this paper? One of the most important conclusions is that it appears very difficult to devise reward systems which are more cost-effective than a simple marginal reward system with a fixed threshold in each period. If the optimal learning of the average student is growing (or declining) over time, it may be desirable to introduce a growth or depreciation factor into the program. Alternatively, the baseline of each student might be deviated from the average score of all students in the class. As long as the individual student cannot affect his or her baseline, however, the learning decision will not depend on how the baseline is updated.

A fixed baseline system also generates the maximum learning gain for a given reward rate. For a discount factor of .9, the learning response is 10 times higher when the baseline is fixed than under any other updating rule in the absence of uncertainty. Because there are no intertemporal trade-offs when the threshold is fixed, students respond to the subsidy as if there were a permanent increase in the payoff to learning.

There is only one instance in which we have been able to demonstrate the possibility of a substantial improvement over the cost-effectiveness of fixed baseline reward systems. This occurs when students believe that there is some probability that the reward system will be terminated next period. Such a belief does not change the student's behavior when the baseline is fixed. However, the student responds differently in reward systems with a trade-off between more learning today and a lower baseline tomorrow. Since the students cannot be certain of obtaining future rewards, they try harder in each period in order to be certain of obtaining the rewards while these are still available. Thus, if students are sufficiently uncertain about the future of the reward system, it is possible to use past test scores to update the student's baseline to increase the cost effectiveness of the reward system.

If it is decided that a reward system should include a baseline updating rule based on past test scores of the individual, then it is important to de-

sign the program in a way that reduces the fluctuations in the learning response. Of the schemes we have considered, our analysis suggests that a system that adjusts the baseline to reflect the previous peak test scores will be most effective in meeting this objective. Unlike systems with immediate updating of thresholds, peak score updating schemes never provide an incentive to the student to reduce learning below the optimal level. If there is no depreciation of the baseline, however, only students with a positive learning growth rate will be rewarded more than once. By introducing a depreciation factor into the baseline updating rule, not only will more students respond to the subsidy, but the learning response of each student will be larger for the same subsidy rates. This effect tends to partially offset the higher reward-learning ratio resulting from the lower baseline.

This initial foray into formal mathematical analysis of the incentive effects of alternative classroom reward systems has produced some important insights. Further work will almost certainly yield more. Since running average updating rules are used in Slavin's cooperative learning programs (Team Assistant Instruction and Student Team Achievement Divisions), careful analysis of their properties is called for. In order to keep the problem tractable, we had to make a number of assumptions that do considerable violence to classroom reality. In our view, the most serious problems lie in the assumptions (1) that there is no error in measuring learning and (2) that the marginal costs of underlearning are comparable to the marginal costs of overlearning (that is, the learning cost function is quadratic). In fact, students can purposely do poorly on a test, therefore, reward systems that lower baselines substantially when a student does very poorly can be easily undermined by strategic behavior. Our conclusion is that reward systems should either not update their baselines or should ignore low scores when baselines are being updated (as the peak period updating system does).

APPENDIX 3-A

THE MODEL

We assume that the objective of a typical student is to choose his or her learning level in each period to maximize the discounted sum of utility. In order to obtain simple closed-form solutions, we assume that ability in any period is a concave function of student learning in that period, L_t, given by $b_{Lt} - (\frac{a}{L_t^*})(L - L_t^*)^2$ where L_t^* is a parameter proportional to the optimal learning in period t and a and b are positive constants. Let w be the cost of learning above baseline $\bar{L}_t$. The the net utility in period t is equal to

$$bL_t - \frac{a}{L_t^*}(L_t - L_t^*)^2 - wL_t + sw \max(0, L_t - L_t).$$

With no loss of generality, we may assume that b = w.[1] Then, if the student discounts utility in each period by a factor of δ, the problem of the student is to choose L in each period to maximize

$$(1) \qquad \sum_{t=0}^{\infty} [-(\frac{a}{L_t^*})(L_t - L_t^*)^2 + sw \max (0, \bar{L}_t - L_t)]\delta^t$$

where $\bar{L}t$ is typically a function of $Lt-1$ and $\bar{L}t-1$.

It is clear from inspection of (1) that in the absence of a reward, the optimal policy of the student is to set L_t^*. Thus, L_t^* may be regarded as the optimal learning during time period t. The utility function has been constructed so that the cost of a given percent deviation from the optimal learning level is proportional to that optimal level. To obtain steady-state solutions, we will assume that L_t^* grows each period by a constant

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[1] If $b \neq w$, let $a' = (a + \frac{b-w}{2})$, $L_t^{*'} = (1 + \frac{b-w}{2a})L_t^*$, and $c_t = L_t^*(b-w)(a+\frac{b-w}{4})$. Then $\frac{a}{L^*}(L_t - L_t^*)^2 + (b-w)L_t = \frac{a'}{L^{*'}}(L_t - L_t^{*'}) + c_t$. Since c_t is a constant, it can be ignored in determining the optimal response of the student.

factor $g(0 < g < 1/\delta)$. We also assume that both the cost of learning and the utility function are unaffected by the reward system.

By appropriately normalizing units and adjusting the value of a, we may set $L_t^*/g^t = L_0^* = 1$. Let $\lambda = g\delta < 1$ be the discount factor normalized for growth and let ℓ_t be the student's learning level in period t, normalized for growth. Then the student's problem may be rewritten as

$$(\max_{\ell_t}) \sum_{t=0}^{\infty} [-a(\ell_t - 1)^2 + sw \max(0, \ell_t - \bar{L}_t/g^t)]\lambda^t.$$

In the absence of the reward system, the student will set $\ell_t = 1$ in each period.

Finally, we may show that at the no-reward optimal learning the elasticity of demand for learning, n, is equal to 1/2a. Ignoring the reward, the single period utility function for the student is $\frac{a}{L^*}(L-L^*)^2 + (b-w)L$. Differentiating with respect to L and setting the resulting expression equal to zero generates the optimal learning level, $L = (\frac{b-w}{2a} + 1)L^*$. Then $n \equiv \frac{-L}{w}\frac{\partial L}{\partial w} = (\frac{b-w}{2a} + 1) \cdot (\frac{1}{2aw})$. Then if $b = w = 1$, $n = 1/2a$.

Appendix 3-B

FORMULAS USED IN TEXT

We present here the general formulas for the present value of the increase in learning and reward costs produced by the various reward systems discussed in the text. These formulas are based on the formal analysis of utility maximizing behavior presented in appendix C (Bishop-Wilson [1980]). The reward-learning ratios presented in table 1 are equal to the reward cost divided by the learning gain. Formulas for distortion ratios are presented but not used to create tables.

1. Fixed Threshold Program: $\bar{L}_t = q\bar{L}_{t-1}$ $(q > 0)$.

Assume that $\bar{L}_0 = qL^*_{-1}$.

(a) If $g \geq q$, the student sets $\ell_t = sn + 1$ for all $t \geq 0$. In this case, we have

Learning gain: $\frac{sn}{1 - \lambda}$

Distortion costs: $\frac{s^2 n/2}{1 - \lambda}$

Subsidy costs: $s(\frac{sn + 1}{1 - \lambda} - \frac{q/g}{1 - \delta q})$

(b) If $g < q$, then there is a $\hat{t}$ such that $\hat{t} = \max \{t:=p^t < g^t(1 + \frac{sn}{2})\}$. Then, $\ell_t = 1 + sn$ if $t \leq \hat{t}$; $\ell_t = 0$ for $t > \hat{t}$. If $p > g(1 + \frac{sn}{2})$, then $\hat{t} < 0$ and there is no response. Otherwise, we have

Learning response: $sn(\frac{1 - \lambda^{\hat{t}+1}}{1 - \lambda})$

Distortion costs: $\frac{s^2 n}{2} (\frac{1 - \lambda^{t+1}}{1 - \lambda})$

Subsidy costs: $s[(sn + 1) \frac{1 - \lambda^{\hat{t}+1}}{1 - \lambda} - (\frac{g}{g})(\frac{1 - (\delta q)^{\hat{t}+1}}{1 - \delta q})]$

2. Yearly Updated Threshold: $\bar{L}_t = pL_{t-1}$ $(p > 0)$.

Define $\bar{g} = p(\frac{sn(2 - p\delta) + 2}{sn(1 - 2p\delta) + 2})$; $\underline{g} = p(\frac{2 - \delta psn}{2 + sn})$. Assume that $\bar{L}_0 = pL^*_{-1}$.

(a) If $\frac{1}{\delta} > g \geq \underline{g}$, then the student sets $\ell_t = (1 - p\delta)sn + 1$. In this case, we have

Learning gain: $(\frac{1 - p\delta}{1 - \lambda})sn$

Distortion costs: $\frac{(1 - p\delta)^2}{1 - \lambda} s^2 n w/2$

Subsidy costs: $s[\frac{(1 - p)^2}{1 - \lambda} sn + 1 - \frac{p}{g}]$ and

(b) for $\bar{g} > g \geq \underline{g}$, the student follows a two period cycle. Learning alternates between $sn + 1$ and $-p\delta sn + 1$. Define $g_0 = \frac{2p}{sn(1 - (p\delta)^2) + 2}$.

(i) If $g > g_0$, then the student responds by first increasing learning (i.e., $\ell_0 = sn + 1$). In this case, we have

Learning gain: $(\frac{1 - \lambda p\delta}{1 - \lambda^2})sn$

Distortion costs: $(\frac{1 + (p\delta)^2}{1 - \lambda^2}) \frac{s^2 n}{2}$

Subsidy costs: $s(\frac{sn(1 + (\lambda p\delta)^2) + 1 - p/g}{1 - \lambda^2})$

(ii) If $g < g_0$, then the student responds by first decreasing learning (i.e., $\ell_0 = -p\delta sn + 1$). In this case, we have

Learning gain: $\frac{\delta(g - p)sn}{1 - \lambda^2} < 0.$

Distortion costs: $\frac{((p\delta)^2 + \ell)}{1 - \lambda^2} \frac{s^2 n}{2}$

Subsidy costs: $\frac{s\lambda[sn1 + (p\delta)^2) + 1 - (p/g)]}{1 - \lambda^2}$

(c) For $g < \underline{g}$, the student does not respond.

3. Generalized Peak Learning Threshold: $\bar{L}_t = q \max (L_{t-1}, \bar{L}_{t-1})$ $(q > 0)$.

Assume that $\bar{L}_0 = qL_{-1}$. As in the text, let n be the probability that the reward will not be discontinued in any given period, given that it has stayed in place up to that period.

(a) If $g > q$, the student will respond by setting $\ell_t = sn(1 - (\pi p\delta)^n)$ for $t = 0,n,2n,\ldots$ and $\ell_t = 1$ in all other periods. The length of the cycle is determined by choosing an n that maximizes

$F(m) = (\frac{1 - p(\pi q\delta)^m}{1 - p(\pi\lambda)^m})(1 - p(\pi q\delta)^m)\frac{sn}{2} + 1)$. In this case, we have

Learning gain: $\frac{(1 - p(\pi q\delta)^n)}{1 - \lambda^n} sn$

Distortion costs: $\frac{(1 - p(\pi q\delta)^n)^2}{1 - \lambda^n} \frac{s^2 n}{2}$

Subsidy costs: $s[(\frac{1 - p(q\delta)^n}{1 - \lambda^n})(sn(1 - p(\pi q\delta)^n) + 1) - \frac{pq}{g}]$

(b) If $0 > g > \frac{2q}{2 + sn}$, the student will respond with $\ell_0 = 1 + sn$ and $\ell_t = 1$ for all $t > 0$. In this case, we have:

Learning gain: sn

Distortion costs: $\frac{s^2 n}{2}$

Subsidy costs: $sn + 1 - \frac{q}{g}$

(c) If $0 < \frac{2q}{2 + sn}$, then there is no response.

APPENDIX A

OBSERVATIONAL DATA COLLECTION

The overall purpose of this study was to develop theoretical frameworks for understanding the influence of high schools on youths' adaptability to workplaces. In concert with the theorization activity, observational-based research addressed questions about instructional methods, contexts of learning, and peer influence in the classrooms. The unit of analysis in this research was the classroom. Our sampling scheme emphasized exemplary programs as opposed to the standard classrooms; it emphasized variation in student motivation among members of the classes; and it emphasized variation in type of classes. This appendix provides a chronology of the observational research and documentation of the resulting data.

Sample Selection and Data Collection Procedures

The observational data collection component of the study involved a pilot site and three exemplary programs.

Pilot Site

During spring 1985, many Columbus-area school districts were contacted and asked to serve as a pilot site. Project staff met with district personnel in some cases and mailed information concerning the project in others. Since it was the end of the school year, many districts were reluctant to participate and oftentimes the district's policies and procedures would have taken longer than our time frame would allow.

The Westerville City School District, a suburban district of Columbus, however, agreed to participate as the pilot or "test" site for this research. The Westerville administration was very responsive, and since there are two high schools in the district, we were able to use the entire project staff for observations without "overloading" one school; also we were able to compare experiences across schools.

The following is a description of Westerville North and Westerville South high schools:

- Westerville City School System
  Westerville North High School
  950 Smothers Road
  Mr. Lee Whitaker, Principal

- Westerville South High School
  303 S. Otterbein Avenue
  Mr. Richard Miller, Principal

South and North high schools are located in Westerville, a growing residential suburb of Columbus, Ohio. North opened in 1975 in response to the rapid growth of the community.

Both schools are comprehensive high schools, but mainly emphasize academic programs. Independent and college placement programs are available to students on a contractual basis. Alternative (gifted and talented) and honors courses are limited in enrollment and offered to students selected on the basis of faculty recommendations and test scores. Each school also offers strong extracurricular programs that include outstanding music and drama departments, a wide variety of clubs and organizations, and highly successful athletic departments.

Surveys indicate that 55-60 percent of the schools' graduating seniors plan to continue their education at 2- and 4-year institutions, 6-9 percent enter the military for further training, and many of the others go into the work force.

- South High School (113 certified staff)

  School Enrollment: 1,945/982 boys; 963 girls

  North High School (93 certified staff)

  School Enrollment: 1,660/827 boys; 833 girls
  93 percent white; 7 percent minority

Classroom observation occurred in 12 separate class periods--2 periods a day for 6 teachers--from Wednesday, May 8, through Monday, May 13. Two classes of English, Chemistry, and Social Studies were observed in each school. To test the methodological question of the appropriateness of 1 or 2 observers, 50 percent of the class periods had 1 observer and 50 percent had 2. All in all, the pilot site involved 72 classroom hours of observation. After the observation period, each teacher was interviewed, a total of 5 students from each of the 12 class periods were interviewd, guidance personnel were interviewed, and the assistant principals for curriculum in each high school were

interviewed. Thus, the sample size for interviews in the pilot study was as follows:

- Administrators 2
- Teachers 6
- Guidance counselors . 2
- Students 60

The interviewing took place from Monday, May 13, through Thursday, May 16.

Following the pilot site observation, the observer/interviewers' notes were coded and typed. The project staff met and discussed the substance of their experiences and addressed various methodological issues to inform the observation process scheduled for fall.

Exemplary Sites

A number of sources of information were used to identify the exemplary classrooms. Teacher publications were reviewed for articles about such programs. Representatives of teacher membership and other professional organizations were contacted. NIE staff and colleagues at other universities and labs and centers were contacted for their suggestions. Finally, nominations for exemplary programs in the area of education and employment that had been submitted to the National Center for a recent audioconference were reviewed.

An explicit criterion for a school or classroom program to be categorized as exemplary included documented evidence on the effectiveness of the programs. We limited our search to nonvocational programs; to high schools with a significant number of college-bound students; large schools with at least 1,500 students; and, in one case, to a school that used cooperative learning techniques.

Table A-1 provides a list of individuals and organizations that were contacted for suggestions of exemplary programs. Table A-2 provides a list of the exemplary program suggestions received. Following are brief descriptions of the three exemplary sites that were chosen.

TABLE A-1

LIST OF INDIVIDUALS AND ORGANIZATIONS CONTACTED
FOR EXEMPLARY PROGRAM SUGGESTIONS

Dr. Helen Hill
Department of Education
Richmond, VA

Mr. William Parrish
National Association of Secondary
School Principals
Reston, VA

Ms. Nona Denton
Illinois Board of Education
Springfield, IL

Mr. Carl Larsen
Iowa Central Community College
Fort Dodge, IA

Dr. Mary Anne Raywid
Chair, Educational Administration
Hofstra University

Dr. Marion Holmes
Executive Director of Career
and Vocational Education
Philadelphia, PA

Dr. John Meerback
Chief, Division of Student Services
Harrisburg, PA

Ms. Marian Craft
Office of Vocational and Adult
Education
U.S. Department of Education
Washington, DC

Ms. Frances Cloyd
Coordinator of Community Relations
and Career Education
Lexington, KY

Dr. Richard Campbell
Director
Cooperative Education/Youth Employment
Lincoln, NE

Dr. Richard Miguel
National Center for Research
in Vocational Education
Columbus, Ohio

Professor Robert Slavin
Johns Hopkins University
Baltimore, Maryland

EBCE Dissemination Project
Berkeley, California

Ms. Pat Schwallie-Giddis
State Coordinator of Career Education
Tallahassee, FL

Dr. Steve Hamilton
Department of Human Development
and Family Studies
Cornell University

Dr. Lanso A. Crim
Superintendent
Atlanta Public Schools
Atlanta, GA

National Child Labor Committee
New York, NY

Dr. Thomas Owens
Senior Research Associate
Northwest Regional Education Lab
Portland, OR

American Association of School
Administrators (AASA)
Arlington, VA

National Alliance of Black School
Educators (NABSE)

National Commission for Cooperative
Education
Boston, MA

Cooperative Education Research Center
Boston, MA

American Vocational Association
Arlington, VA

Council of the Great City
Schools (CGCS)
Washington, DC

Dr. Ronald Bucknam
National Institute of Education
Washington, DC

Mr. Don S. Ayers
Supervisor, Secondary Guidance
Programs
Virginia Department of Education

Association for Supervision and
Curriculum Development (ASCD)
Alexandria, VA 22314

TABLE A-2
EXEMPLARY PROGRAM NOMINATIONS

School	Source of Recommendation	Program Description
New Garden Friends School Greensboro, N. Carolina	Mary Anne Raywid	This small private school was established on the principles of Friends belief in the worth of the individual. Students proceed at their own pace and are taught to be independent.
Metro Secondary School Cedar Rapids, Iowa	ASCD	This high school offers a special program in career-education/youth employment.
Linn-Mar High School Marion, Iowa	ASCD	This high school offers a special program in career-education/youth employment.
J.F. Kennedy High School Bloomington, Minnesota	ASCD	This high school offers a special program in career-education/youth employment.
Parkway North High Creve Coeur, Missouri	ASCD	This high school offers a special program in career-education/youth employment.
Boise 70001 Project Boise, Idaho	Robert Gilbert	Boise 70001 is part of a national nonprofit corporation that helps 16 to 21-year-old high school dropouts. It is a program of employment, training, education, and motivation.
North Central High Indianapolis, Indiana	Audioconference Nomination	"Learning Unlimited" offers a community-based career exploration program. Students' work is individualized and contracted with both teachers and parents.
Highland High School Salt Lake City, Utah	Audioconference Nomination	This voluntary alternative EBCE program uses the entire community as a school. The program focuses on direct experience learning in a community setting where students apply classroom theory to solve practical problems.

Table A-2--Continued

School	Source of Recommendation	Program Description
Waverly Public Schools Waverly, Nebraska	Richard Campbell Nebraska Dept. of Education	This program involves the teaching of employability skills, on-the-job training in specific occupations, integration of handicapped, disadvantaged and regular students into one program.
Olney High School Philadelphia, PA	Tom Owens	The Academy for Career Education offers a comprehensive program responsive to students academic, personal, and vocational development.
Tigard High School Tigard, Oregon	Tom Owens	This high school offers an outstanding program in career exploration through community experience.
Cincinnati School for the Creative and Performing Arts Cincinnati, Ohio	Kathy Borman	SCPA provides an educational environment that makes the arts the center of student attention. All students must audition before the school faculty for admittance.
Las Cruces Public Schools Las Cruces, New Mexico	Tom Owens	EXCEL-the career program is an experience-based career education (EBCE), designed to explore careers in the community while earning school credit.
Southside Senior High The Greenhouse School Rockville Center, New York	Mary Anne Raywid	
Jefferson County Open H.S. Evergreen, Colorado	Audioconference Nomination	This is a small public alternative school with a Walkabout-based curriculum that is experiential, challenging, ungraded, and individualized.

Table A-2--Continued

School	Source of Recommendation	Program Description
Mt. Hebron High School Ellicott City, MD	John Bishop	The math department has had extensive involvement with cooperative learning; it is now being introduced to other academic departments.
The Village School Great Neck Schools Great Neck, New York	Mary Anne Raywid	
Philadelphia Job Search John F. Kennedy Center Philadelphia, PA	Marion Holmes	This program operates in several high schools to bridge the gap between school and work by providing students with important skill needed to acquire a job now and throughout their lives.
Murry Bergtraum H.S. for Business Careers New York, New York	Committe for Economic Development Report	Bergtraum provides a strong business careers orientation; students are exposed to a variety of opportunities for both cultural enrichment and social involvement.
BOCES Yorktown Hts., NY	Peter Copen	Walkabout Program emphasizes a career preparation and a 9-week career internship.
Gloucester High School Gloucester, Virginia	Don S. Ayers	This school has an exemplary "youth employability program, the Gloucester-Mathews Job Referral Service.
Fayette Co. Public Schools Lexington, KY	Ron Bucknam	The Fayette County Schools offer an experienced-based career education program, operating in four high schools that combines "classroom" learning with "real world" experiences.

Table A-2--Continued

School	Source of Recommendation	Program Description
Linworth Alternative H.S. of Worthington High Worthington, Ohio	Richard Miguel	In a highly competitive college prep high school the alternative high school provides students the option to plan their curriculum based on their particular learning needs.
Jefferson High School Portland, Oregon	Audioconference Nomination	The Financial Services Academy is a unique industry-school partnership combining practical training related to financial services with basic academic studies.

(1)

Mount Hebron High School
9440 Route 99, Old Frederick Road
Ellicott City, MD 21043
James R. McCrumb, Principal

Mt. Hebron was selected as an exemplary site for its extensive use of cooperative learning techniques, used largely in the math department. Mount Hebron is located outside of Ellicott City in central Maryland. The Mount Hebron faculty is composed of 47 full-time and 4 part-time teachers. In addition, Mount Hebron has three administrators, three guidance counselors, and two media specialists.

Mount Hebron is a comprehensive high school that includes grades 9-12. The majority of courses are academically oriented and include advanced placement offerings in English, social studies (European and U.S. History), science (biology, chemistry and physics), and French. The math department offers three levels of calculus (through differential equations) as well as computer programming (basic and pascal). In addition, all academic departments have developed underclass honors and/or Gifted and Talented Programs. Mount Hebron also has a well-developed business curriculum within the school and a vocational program to which our students are bussed.

Each year approximately 75-80% of graduates continue their education at postsecondary institutions. Of the 220 members of the Class of 1985, 54.5 percent entered a 4-year college; 22 percent a 2-year college; and 2 percent a trade, technical, business, or nursing school. Almost all of the rest were able to obtain entry-level jobs (17 percent) or enter the armed services (2.5 percent).

School Enrollment: 1,004/489 boys; 515 girls
4 percent Asian; 6 percent black; 90 percent white

Observations at Mt. Hebron were centerd in the math department, where cooperative learning techniques were being used. A total of nine classes were observed--three classes each for three algebra teachers (2 classes used cooperative learning and one not using cooperative learning). Additionally, five students from each class, the three teachers, a guidance counselor, and the principal were interviewed. Observations began on Wednesday, October 23d and were conducted for 4 consecutive school days. Interviews were conducted over the period Monday, October 28 through Thursday, October 31.

(2) Cincinnati School for the Creative
and Performing Arts
1310 Sycamore Street
Cincinnati, OH 45210
Mr. William Dickinson, Principal

Cincinnati's innovative School for the Creative and Performing Arts (SCPA) operates from a philosophy that combines elitism with a concern about the arts. The arts are viewed as a means of achieving racial integration, with the means being as important as the end. Elitism is essential because the school recognizes and promotes the best talent through its requirement that children are only admitted after a successful audition before the school faculty and through its selection of only the best talent for public performances and recognition. The grade range in the school is from the 4th to the 12th grades.

Though each student is involved in a college preparatory curriculum, all students are required to take at least one class in each major arts area. The school defines these major arts as dance, drama, instrumental music, and visual arts. In addition, all students must major in one area of the arts such as creative writing, dance, drama, instrumental music, music composition, vocal music, visual art, fine arts, music theater, and stagecraft.

SCPA has an annual average of 74 percent of its graduates who pursue higher education at a 4 year institution and another 15 percent continue at a 2-year or technical institution.

School Enrollment: 1,120 students/467 boys; 653 girls
45 percent black; 52 percent white; and 3 other

Classroom observations of two classes of Psychology, History, and Drama Lab were conducted from Wednesday, September 18th, through Monday, September 23d at the Cincinnati School for the Creative and Performing Arts. During the week of September 23d through September 30th, the principal, the teachers of the classes observed, and five students from each class were interviewed.

(3) Murry Bergtraum High School
411 Pearl Street
New York, NY 10038
Ms. Barbara Christen, Principal

Murry Bergtraum High School for Business Careers is a coeducational secondary school located in the City Hall area of downtown New York City. Because of Bergtraum's proximity to the Manhattan Civic Center, financial district, and institutions of higher education, students are exposed to an extraordinary variety of opportunities for both cultural enrichment and social involvement.

Because of Murry Bergtraum's strong business careers orientation, the school attracts in excess of 25,000 applications each year. Several factors are carefully considered in the selection process, including past school records and reading and mathematics scores. In order to ensure diversity among entering students, consideration is also given to ethnic background and residential location. Approximately 700-750 students are admitted to the school each year.

Murry Bergtraum High School has a diverse student population. There are students at the school from each borough in the city: 38 percent of the students are from Brooklyn, 31 percent from Manhattan, 19 percent from the Bronx, 10 percent from Queens, and 2 percent from Richmond. Eighty-five percent of the students attended public elementary and intermediate schools. The others have come from private and parochial schools located throughout the five boroughs.

Students are required to complete a program of college preparatory studies with a strong emphasis in business education and skills training. Their preparation includes 4 years of study in English, 3 years of social studies; 2 years in laboratory science; at least 2 years in mathematics; a year or more of computer languages or foreign languages; typewriting, and aesthetic arts; and 3 or more years of study in 1 of the following "concentrated" areas of interest:

- Accounting
- Computer Science
- Marketing
- Secretarial Sciences
- Securities and Finance

Field-Work Internship experience is required of all graduates. College acceptance rates are among the highest in the city; 80 percent of its graduates choose to further their education immediately; many others work for a year or 2, then return to higher education.

School Enrollment: 2,943/885 boys; 2,058 girls
80 percent black and Hispanic; 12 percent Oriental; 8 percent other

The observations at Murry Bergtraum occurred in two classes of English, economics, computer science, and secretarial studies. Our project team of three members began observation on Wednesday, October 16th, and observed 4 consecutive school days concluding on Monday, October 21st. The team then interviewed students (five from each class), each of the teachers observed, the principal, and the school's advisory board chairman. All interviews were completed October 21st through October 25th.

Data Coding and Storage

A considerable quantity of data was collected throughout the course of the project. These data were of a qualitative nature being comprised of observer reports of classroom events and comments made by students, teachers, or administrators during the course of (unstructured) interviews. The data were entered onto an IBM PC and formated for analysis by a text retrieval program entitled SUPERFILE.

All of the data were coded by the field staff using the codes listed in table A-3. For example, portions of the interview or classroom events that were relevant to the subject--collaboration among students on homework--were coded 14-00. Many of the data entries had multiple codes assigned to them. The purpose of the codes were to serve as keywords for SUPERFILE. Additional keywords included class subject, interviewer name, school, interview type (student, teacher, and so forth), and class level (see figure A-1). To get a listing of all comments by Seniors at Cincinnati's School for Creative and Performing Arts, we would request SUPERFILE to retrieve data entries with the keywords 43-00 (higher education plans), SR (for seniors), and SCPA (for the high school name).

TABLE A-3

TOPIC AREAS AND
DATA COLLECTION CATEGORIES
FOR INTERVIEWS AND OBSERVATIONS

I. Relating to Others

1-00 Qualities or achievements essential to be in leading crowd
2-00 Qualities or achievements essential for:
- 2-21 - peer respect
- 2-22 - school administrator respect
- 2-23 - teacher respect
- 2-24 - guidance counselor respect
- 2-25 - parents respect
- 2-26 - respect of leading crowd
- 2-27 - importance placed on each of the above groups
- 2-28 - getting a good job
- 2-29 - promotion on the job

3-00 Friends in school
4-00 Friends career plans
5-00 Subgroups in the school
- 5-21 - share of friends in various groups
- 5-22 - vocational students
- 5-23 - socializing and conferring in other subgroups

6-00 Parental career expectations

II. Homework

7-00 Extent of assignments
8-00 Nature of assignments
9-00 Student interest level
- 9-01 - Student choices on assignment

10-00 Extent homework is completed
11-00 Extent of help students get on homework
- 11-21 - parents
- 11-22 - students
- 11-23 - teachers
- 11-24 - others
- 11-25 - rules and expectations
- 11-26 - actual practice

12-00 Procedure for grading homework
13-00 Rewards/penalties for doing homework
14-00 Collaboration among students

III. Grading

15-00 Factors influencing grades
16-00 Relative importance of factors influencing grades
17-00 Extent to which a curve determines grades
18-00 Extent to which grades are public knowledge
19-00 Value placed on grades by students
- 19-01 - Feelings about tests
- 19-02 - Feelings about grades

20-00 Parental influence on grades
21-00 Peer influence/respect concerning grades
22-00 Grading system fairness
- 22-01 - Cheating

IV. Group Work

23-00 Structure of group work
24-00 Typical composition of group members
25-00 Duration of group
26-00 Strategies used to deal with group dropouts
27-00 Strategies used to deal with group standouts
28-00 Visibility of individual student contribution
29-00 Individual sharing of group responsibilities
30-00 Motivation derived from fellow students
31-00 Rewards for group effort
32-00 Distribution of rewards
33-00 Development of group norms

V. Classroom Recognition

34-00 Chances for recognition
35-00 Kinds of recognition/rewards
36-00 Basis for giving recognition/reward
37-00 Level of competition among students
38-00 Peer reaction to recognition
39-00 Qualities of a student that teachers feel are important
  39-21 - tries hard
  39-22 - improvement during year
  39-23 - knowledge of material
  39-24 - follows directions
  39-25 - helps others in class
  39-26 - very smart
  39-27 - interaction and participation
  39-28 - self-discipline
40-00 Effects of putting additional effort into studies
  40-21 - ways teachers learn about it
  40-22 - consequences

VI. After Completing School

41-00 Determinants of success in jobs after completing school
  41-21 - types of jobs
42-00 Determinants of success in jobs recently and/or currently held
  42-21 - types of jobs
43-00 Higher education plans

VII. Locus of Control

44-00 Impressions about ability to control what happens to them in the future
45-00 Ability to control what happened to them while in school
46-00 Control of ability to get a good job

VIII. Classroom Environment

47-00 Rules and expectations
- 47-21 - Appearance of classroom
- 47-22 - Appearance of teacher
- 47-23 - Classroom decisionmaking
- 47-24 - Appearance of students

48-00 Attentivenes
49-00 Discipline
50-00 Student participation
51-00 Attendance tardiness
- 51-01 - Maintenance

IX. School/Curriculum Environment

52-00 School-wide recognition, e.g., honor society, PTO recognition
53-00 Individuals' choice of classes
- 53-21 - when choices were made
- 53-22 - why choices were made
- 53-23 - who influences choices
- 53-24 - advanced classes

X. Other

54-00 Teacher actions/unique instructional strategies
- 54-01 - Time on task
- 54-02 - Interacting with observer
- 54-03 - Teacher conferring with other teachers
- 54-04 - Time off task (other subjects)
- 54-05 - Time off task
- 54-06 - Teacher planning
- 54-07 - Teacher conferring with students

61-00 Student presentations
62-00 Quality of work/discussion
63-00 School cohesiveness
- 63-01 - Importance in classroom
- 63-02 - Importance to students
- 63-03 - Teacher mentions unity
- 63-04 - Teacher informal interaction with students

64-00 Racial issues
- 64-01 - Feelings about prejudice
- 64-02 - Teacher mention of prejudice

65-00 Mention of college
- 65-01 - Teacher relating class events to college
- 65-02 - Students relating his/her effort to college

66-00 Personal qualities of student
67-00 Motivation
- 67-01 - Students feelings about it
- 67-02 - Reasons given
- 67-03 - Effects of motivation

68-00 T-S interaction
- 68-21 - Teacher encouragement
- 68-22 - Teacher initiating student leadership
- 68-23 - Teacher showing trust
- 68-24 - Teacher respect
- 68-25 - Teacher becoming involved in personal problems
- 68-26 - Teacher initiates humor

69-00 Risk taking

70-00 Self-esteem
- 70-21 - Teacher praises student achievement
- 70-22 - Teacher praise (general)
- 70-23 - Praise from peers
- 70-24 - Student acknowledgement of teacher praise
- 70-25 - Negative comments from teacher
- 70-26 - Negative comments from one student to other students

Observation Note Format for Keywords:

Code 1/Code 2/. . ./Observer/School/O/Class Name

Interview Comment Format for Keywords:

Code 1/Code 2/. . ./Interviewer/School/I/Respondent Type

where Code i = codes from table A-3; variable number of codes per entry

Observer (Interviewer) = NCRVE Staff Member initials

School = WN - Westerville North
WS - Westerville South
SCPA - Cincinnati School for Creative and Performing Arts
MB - Murry Bergtraum (NYC)
MHEB - Mt. Hebron High School

Class Name = CHEM - Chemistry (Westerville)
CS - Computer Science (Murry Bergtraum
DRAMA - Drama Lab (SCPA)
ENG - English (Westerville Bergtraum)
GERM - German (Mt. Hebron)
GOVT - American Government (Westerville)
HIS - History
MATH - Mathematics (Mt. Hebron)
PSYCH - Psychology (SCPA)
PHYS - Physics (SCPA)
SECR - Secretarial Science (Bergtraum)

Respondent Type = SOPH - Sophomore
JR - Junior
SR - Senior
UNK - Grade level unknown
TEACH - Teacher
PRIN - Administrator
GC - Guidance Counselor

Figure A-1. SUPERFILE keywords other than topic areas and data collection categories

APPENDIX B

FAIR DISTRIBUTION RULE IN A COOPERATIVE ENTERPRISE

Suk Kang

This appendix considers a fair distribution rule in a cooperative enterprise such as a team of students participating in a cooperative learning situation in which rewards go to the group rather than the individual. It is assumed that (1) N agents are participating, (2) the total output of the enterprise is a function of the sum of nonnegative efforts contributed by the N agents, (3) the distribution of the output is determined by each agent's relative effort level, and (4) the effort level of each agent is observed completely by the other agents.

It is postulated that the fair distribution rule needs to satisfy the three conditions. Denoting the effort level by the ith agent by E_i and the share of the output by S_i, the conditions are written as

(1) Completeness

$$\sum_{j=1}^{N} S_j = 1 \text{ for all } E_i,\ i=1,..,N$$

(2) Symmetry

$$S_i = f(E_i, E^i) \text{ where } E^i = \sum_{j \neq i} E_j$$

(3) Homogeneity

$$f(aE_i, aE^i) = f(E_i, E^i) \text{ for } a > 0,$$

where a is a positive constant.

Equation (1) expresses the requirement that output is completely distributed among all the agents; (2) expresses the condition that every agent is rewarded according to the same rule, that is, the share of the ith agent is a function of the effort level of the ith agent and the sum of the efforts by all

the other agents; and (3) expresses the condition that a proportionate increase in efforts does not change shares.

One rule that satisfies conditions (1), (2), and (3) is the one considered by Sen (1966):

(S) $S_i = c(E_i/E) + (1-c)(1/N)$

where E is the total effort, $E = \sum_{j=1}^{N} E_j$, and c is a constant.

The rule (S) covers the complete merit rule, $S_i = E_i/E$; the complete egalitarian rule, $S_i = 1/N$; and the intermediate cases between the two.[1]

It shall be shown that the rule (S) completely characterizes the fair distribution rule when the number of agents is more than three and the function f is differentiable.

Theorem

When f is differentiable and $N \geq 3$, the distribution rule (S) is the only rule that satisfies the conditions (1), (2), and (3).

Proof

Conditions (2) and (3) imply that the share S_i can be written as $S_i = g(t_i)$, where t_i is the ith agent's share of effort, $t_i = E_i/E$. Condition i then implies that $S_i = 1/N$ when $t_i = 1/N$, that is, when all the agents expend equal effort, the output is evenly distributed.

The share of the ith agent is written

(1) $S_i = g(t_i) = 1/N + h(t_i)$.

Condition (1) requires

(2) $\sum_{j=1}^{N} h(t_j) = 0$ for all $\underline{t} = (t_1,\ldots,t_N)$, $t_i \geq 0$, $\Sigma t_i = 1$

---

[1]Using the distribution rule (S), Sen (1966) derived the optimum value of the weight c in the symmetric Nash-type equilibrium. Under the assumption that total output is a function of total effort, the optimal weight is given by the elasticity of output with respect to effort level.

and

$$(3)\quad \sum_{j=1}^{N} \partial h(t_j)/\partial t_i = 0 \text{ for } i = 1,..,N.$$

Equation (3) can be rewritten as

$$(4)\quad \sum_{j=1}^{N} h'(t_j)t_j = h'(t_i) \quad i = 1,..,N,$$

where h' is the derivative of h. Equation (4) implies

$$(5)\quad h'(t_i) = h'(t_j) \text{ for all } t_i \text{ and } t_j,\ i,j = 1,\ldots,N,\ \Sigma t_j = 1.$$

It is straightforward to verify that when $N \geq 3$, the first derivative of h is constant, and so h is linear in its argument. One can write $h(t_j)$ as

$$(6)\quad h(t_j) = dt_j + e,$$

where d and e are unknown constants, and solve for d and e.

Substitution of (6) into (2) yields

$$(7)\quad d \sum_{j=1}^{N} t_j + Ne = d + Ne = 0,$$

Then,

$$g(t_i) = 1/N + dt_i - d\ (1/N) = (1-d)\ 1/N + dt_i.$$

Q.E.D.

Remark 1

When N = 2, equation (4) is reduced to $h'(t_1) = -h'(1-t_i)$; therefore, any function that has the property $h(t_i) + h(1-t_i) = 0$ satisfies the fairness conditions. This is because when N = 2, there is only one degree of freedom in the shares of effort.

Remark 2

The theorem does not necessarily imply $c > 0$. That is, more effort by the ith agent, when the effort by others is constant, may result in less share by agent i. For an economically meaningful and fair rule, c should be restricted

to be nonnegative. Furthermore, it may be reasonable to impose the condition that when effort level of the ith agent is nonnegative the ith share is also nonnegative. To satisfy this requirement c needs to be less than or equal to one. Thus, the requirements of fairness considerably restrict the form of distribution rules.

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APPENDIX C

INCENTIVE SCHEME AND DETERMINATION OF EFFORT IN COOPERATIVE ORGANIZATIONS

Suk Kang

This appendix considers the relationship between incentive scheme and effort level by the agents in a cooperative enterprise. The term Enterprise may represent a cooperative learning team, co-op, labor-managed firm, or any other organization in which total output of the production unit is determined by the effort level of agents, where the marginal productivity of the agent depends on other agents' effort level, and where output is completely distributed among the agents. The problem addressed in this appendix is that of determining the distribution rule that induces agents to choose the optimal level of effort.

It is posited that (1) the welfare of the organization is measured by the unit of effort, (2) each agent voluntarily chooses his or her effort level, and (3) the individual agents' effort levels are visible to all the agents. Furthermore, it is postulated that there is agreement among the participating agents that the distribution of the output should be determined based on the individual agent's effort level and that the distribution rule must be fair. (See appendix B).

In the neoclassical theory of distribution, workers get paid their marginal contribution to output, and the profit (surplus) is taken by the owner of the capital. However, in the cooperative production model, the total product needs to be fully distributed among the agents so that when the production function is not homogeneous of degree one in effort, the neoclassical distribution rule will not be a solution to our problem.

In deriving the optimal distribution rule, I will consider a special case in which total output is determined by the sum of effort by the agents: The N

agents are identical, and the production function is in Cobb-Douglas form. Denoting the effort level of the ith agent by E_i, total effort by E, and total output by Y, the total output is written as

(1) $Y = E^{\beta}$, where $\sum_{i=1}^{N} E_i = 1$ and $0 < \beta$

The share (ratio) of the ith agent to total output is determined by the ith agent's effort level and the sum of the all other agents' effort. Limiting the distribution rule to the one that satisfies general requirements of fairness (see appendix B), the share of the ith agent is written as

(2) $S_i = (1 - \alpha)\ 1/N + \alpha(E_i/E)$. $0 < \alpha$.

Equation (2) may be interpreted in the following manner. The distribution rule is a weighted average of the egalitarian rule, $S_i = 1/N$, and the complete merit rule, $S_i = E_i/E$. It is assumed that the marginal cost of effort perceived by the agent is constant and common across the agents. Given technology (1) and distribution rule (2), the objective function of the ith agent is written as

(3) $S_iY - cE_i$,

where c is the marginal cost of effort, $c > 0$. Differentiating (3) with repsect to E_i, the first-order condition for the maximum is given by

(4) $\alpha E^{\beta-1} + \alpha(\beta-1)E_iE^{\beta-2} + (1-\alpha)/N \times \beta E^{\beta-1} = c$.

Since, by assumption, the agents are identical, the equilibrium level of effort must be symmetric, that is, the optimal levels of effort are equal across all the agents.

Denoting the equilibrium value of effort by E^*, substitution of E^* into (4) yields

(5) $E^* = [cN/(\alpha(N-1)+\beta)]^{1/(\beta-1)}/N$

Hence, the equilibrium value of effort is (1) decreasing in the marginal cost of effort (c), (2) increasing in α (see [2]), and (3) decreasing in the number of agents (N).

We consider the optimal distribution rule given the technology and the number of agents. Equation (5) shows that by raising " " effort level, the total output will increase. However, more output does not necessarily imply higher level of social welfare. This is because, at the higher level of effort, the marginal return from extra effort, which is expressed as the increment of output, may be less than the marginal cost of effort.

In order to derive the optimal level of effort, the social welfare of the organization at a symmetric equilibrium is written as

$$(6) \quad V = E^{\beta} - cE = (NE^*)^{\beta} - cNE^*$$

where E^* is the value of effort corresponds to a symmetric equilibrium. The optimal value of E^* is then given by the following equation:

$$(7) \quad E^* = (c/\beta)^{1/(\beta-1)}/N.$$

Equating (7) and (5), we can see that the optimal distribution rule must satisfy

$$(8) \quad \alpha = \beta$$

Note that β is the elasticity of output with respect to effort and the distribution rule can be written as

$$(9) \quad S_i = [(Y/E - \partial Y/\partial E)\, E/N + (\partial Y/\partial E)E_i]/Y$$

or

$$(10) \quad S_i Y = (Y/E - (\partial Y/\partial E))E/N + (\partial Y/\partial E)E_i.$$

Equation (10) implies that ith agent's share (in the unit of output) is the sum of the two components. The first is the marginal product multiplied by the effort level (see the second term of [9]), and the second is the profit per worker, with profit being the output that is not distributed according to the individual agents' effort level. Although the optimal distribution rule in (10) is derived for the Cobb-Douglas case, it can be shown that the rule (10) applies to the general functional forms. (Miyazaki 1979).

Condition (8) implies that if the distribution rule is such that $\alpha < \beta$, agents underinvest their effort, and if $\alpha > \beta$, overinvestment of effort occurs.

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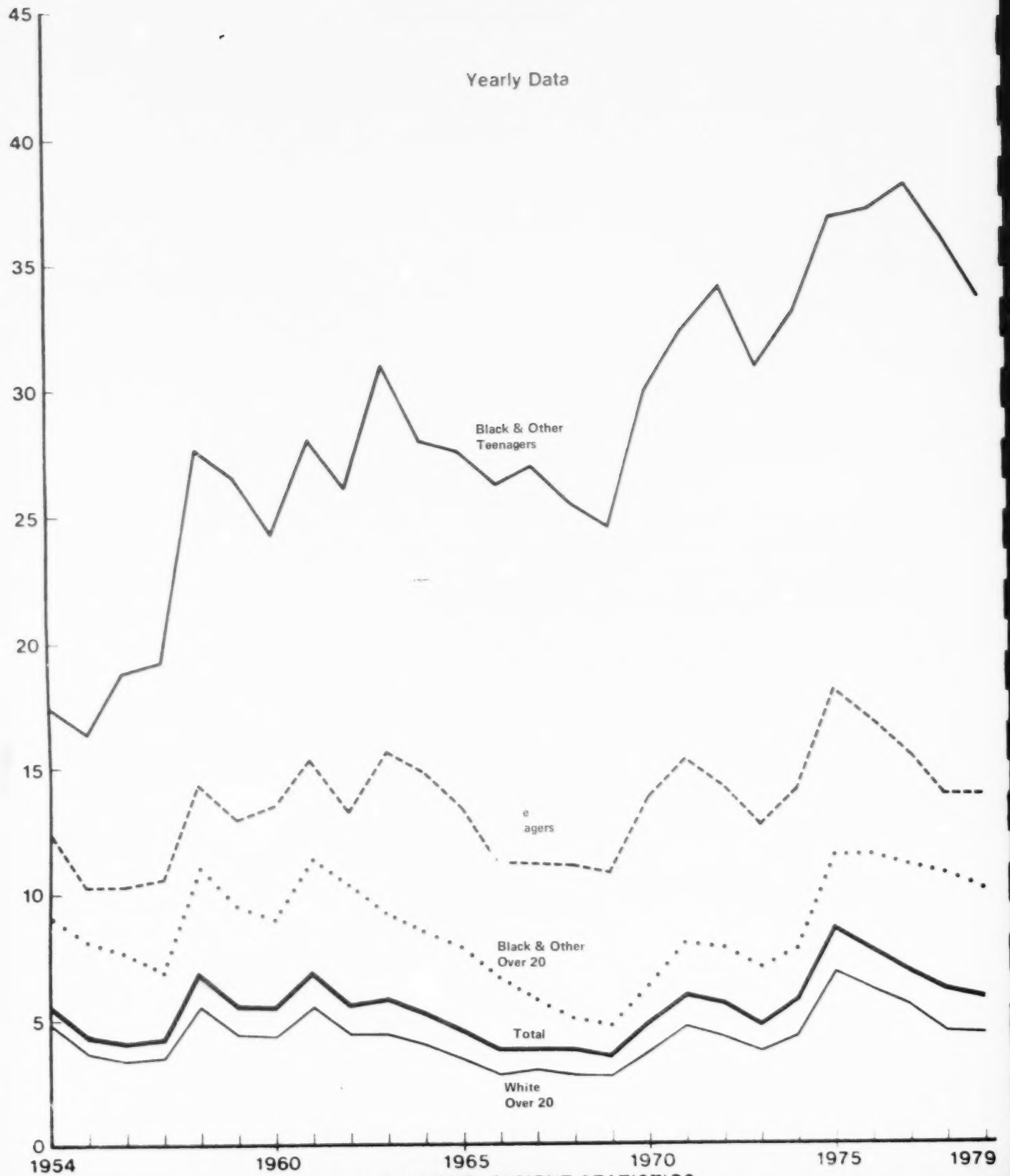

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